Iterative Equalisation and Forward Error Correction

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Outline

- FEC, Viterbi Equalisation and Channel Capacity
- Turbo Equalisation (TE)
- Low-Density Parity-Check (LDPC) Codes and TE
- Equalisation and FEC in OFDM Systems
Fibre-Optical Transmission System

- Chromatic dispersion (CD)
  group velocity dispersion (GVD)

- Polarisation mode dispersion (PMD)
  differential group delay (DGD)

- Amplified spontaneous emission (ASE) noise and other noise

- Self phase modulation (SPM) and cross phase modulation (XPM) noise

10 Gb/s…100 Gb/s
\[ BER < 10^{-16} \]
**Forward Error Correction (FEC) Coding**

Net coding gain (NCG) for code rate $R=0.93$ @ $BER=10^{-16}$

First generation FEC:
- $RS(255,239)$: NCG = 6.3 dB
- $RS(2720,2550)$: NCG = 8.2 dB

Concatenated codes:
- $RS(255,245) \times RS(246, 240)$: NCG = 9.0 dB
- $BCH(2040, 1930) \times BCH(3860,3824)$: NCG = 9.3 dB

LDPC codes:
- $LDPC(32640,30595)$, hard, bit-flip: NCG = 8.2 dB
- $LDPC (32640,30595)$, soft, Sum Prod. Alg.: NCG = 9.7 dB

**FEC performance, R=0.93, AWGN channel**

![Graph showing the performance of different FEC codes]
System Concept I

**Standard equalisation and decoding:** separated FEC decoding and equalisation for channel dispersion

- suboptimum receiver

![System Concept Diagram]
**System Concept II**

**Standard equalisation and decoding:** separated FEC decoding and equalisation for channel dispersion

- suboptimum receiver

**Optimum equalisation and decoding:** over-all maximum-likelihood decoding of combined FEC and dispersion code

- too complex for implementation at 10 Gb/s…100 Gb/s and $BER < 10^{-16}$
**System Concept III**

**Standard equalisation and decoding**: separated FEC decoding and equalisation for channel dispersion
- suboptimum receiver

**Optimum equalisation and decoding**: over-all maximum-likelihood decoding of combined FEC and dispersion code
- too complex for implementation at 10 Gb/s…100 Gb/s and $BER < 10^{-16}$

**Approach**: 
- iterative equalisation and decoding = *Turbo Equalisation* (TE)
  - punctured convolutional code (CC)
  - low-density parity-check (LDPC) code
- demanding for $BER << 10^{-4}$
**System Concept IV**

**Standard equalisation and decoding**: separated FEC decoding and equalisation for channel dispersion
- suboptimum receiver

**Optimum equalisation and decoding**: over-all maximum-likelihood decoding of combined FEC and dispersion code
- too complex for implementation at 10 Gb/s…100 Gb/s and $BER < 10^{-16}$

**Approach**:
- iterative equalisation and decoding = *Turbo Equalisation* (TE)
  - punctured convolutional code (CC)
  - low-density parity-check (LDPC) code
- TE with outer Reed-Solomon (RS) code

![Diagram of system concept IV]

- Outer code rate $R_{RS} \approx 0.93$
- Inner code rate $R_{TE} \approx 0.97$
- Total code rate $R = R_{TE} \cdot R_{RS} \approx 0.9$
Performance Bounds of a Simplified DGD Channel Model

- Linear approximation of DGD channel, $0 \leq \Delta \tau / T \leq 2$: impulse response ($T$ spaced) according to [*] (worst-case channel model)
  \[ h = \left[ \frac{1}{2} \cos^2 \left( \frac{\pi}{4} \left( 2 + \frac{\Delta \tau}{T} \right) \right), \cos^2 \left( \frac{\pi}{4} \frac{\Delta \tau}{T} \right), \frac{1}{2} \cos^2 \left( \frac{\pi}{4} \left( 2 + \frac{\Delta \tau}{T} \right) \right) \right] \]
  with chi-square noise

[*] Jäger, Speidel, Bülow et al., JLT, vol. 24, pp.1226-1235, Mar. 2006

- Mutual information $I(X;Y)$ to calculate performance bounds independent of equaliser implementation

- Channel Coding Theorem: for quasi error free decoding, an FEC code is required with overall code rate $R \leq I(X;Y)$

\[ h(t, \Delta \tau) \]

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OSNR penalty normalised to $\Delta \tau = 0$, $I(X;Y) = 0.9$

unachievable penalty region due to channel capacity limits for NRZ signaling and direct detected DGD channels
Viterbi Equalisation Using Outer FEC Code

Maximum Likelihood Sequence Estimation (MLSE)
= Viterbi equaliser (VE)

- Probabilistic channel model used for metric calculation
  \[ f(r(t) | a_1, ..., r(t) | a_{2L}) \]
- Channel estimation by adaptive histogram methods

- Outer RS(255,239) code for BER=10^{-16}
- VE (4 states), \( q=4 \) bit ADC, \( T \)-spaced log-likelihood lookup-table, histogram-based adaptation
- Gap to mutual information bound: 6 dB
Turbo Equalisation with Punctured Convolutional Code (CC)

- Soft-input / soft-output maximum a-posteriori symbol-by-symbol estimator (MAP-SSE)
- Feedback of extrinsic information $\Lambda$ (estimated symbol reliabilities excluding a-priori information)
- Remove correlations: interleaver $\Pi$, deinterleaver $\Pi^{-1}$
  = pseudo-random permutation of bit positions

Convergence of iterations

OSNR penalty normalised to $\Delta \tau = 0$, $I(X;Y) = 0.9$
TE with Low-Density Parity-Check (LDPC) Codes

- Block code defined by sparse parity-check matrix or by Tanner graph
- Near Shannon limit performance for large codes ($n >> 1000$)
- Soft-decision soft-input decoding algorithms: “Belief Propagation”, “Sum-Product”,...
- Arbitrary code rate, code design tools, efficient encoders,...
- LDPC codes combined with equalisation → TE

Gap to Capacity Limit

Typical OSNR gap to channel capacity

- MLSE + RS(255,247): \(\leq 4.5 \ldots 6.1\) dB
- CC-TE + RS(255,247): 1.8 \ldots 2.8 dB
- LDPC-coded TE + RS(255,247): 0.9 \ldots 1.5 dB
- LDPC-coded TE + RS(2720,2636): 0.7 \ldots 1.3 dB
**Approach: Coherent Detection and OFDM**

**Optical Orthogonal Frequency Division Multiplexing (OFDM)**

- Parallel transmission by a large number of OFDM sub-carriers (64....1024)
- Coherent system; frequency and channel estimation by pilot symbols
- Multi-level modulation (M - QAM)
- Polarisation multiplex (PolMUX)
MIMO Optical OFDM and Equalisation

- Polarisation multiplex = 2 x 2 multiple input / multiple output (MIMO) system
- Bit interleaved coded modulation (BICM) on OFDM sub-carriers

- PMD: MIMO receiver = 2x2 matrix per sub-carrier
- CD: DCF, OFDM pre-distortion or OFDM equaliser
- QAM: Iterative demapping and decoding of BICM
- Performance limitation by SPM/XPM and ASE noise, loss due to guard interval
Conclusions

- Channel capacity constraints: OSNR penalty
- TE outperforms Viterbi equalisation and decoding
- About 1 dB gap to mutual information limit

- Approach to decrease mutual information limit:
  - coherent technique
  - multi-level modulation
  - polarisation multiplex
  - optical OFDM

- References: