On the interaction between network coding and the physical layer

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Collaborators

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- Caltech: Michelle Effros
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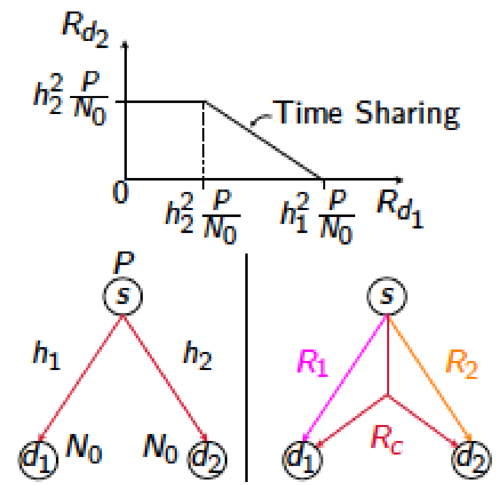
- Stanford: Andrea Goldsmith, Ivana Maric (now Ericsson)
- Rutgers: Ivan Seskar
- BBN: Abhimanyu Gosain

Regimes of SNR

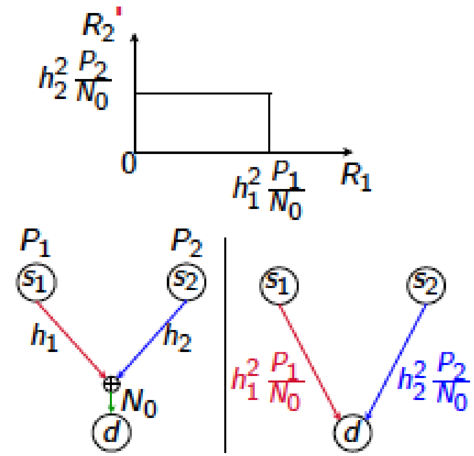
- Information theoretic sinights: we consider three regimes:
  - Low SNR: noise limited
    - Separate PHY coding from network coding
  - High SNR: interference limited
    - Analog network coding
    - Discrete approximations
  - Other SNRs:
    - Use equivalence theory for bounds
- Practical considerations:
    - Consider insight from separation to provide practical approaches – WIMAX case study
    - Low-power chip
- Joint PHY and network coding may be limited in usefulness

Low-SNR Approximation

- Broadcast:
  - Superposition coding rates ~ time-sharing rates
  - Common rate received by both destinations rate received only by the most reliable destination

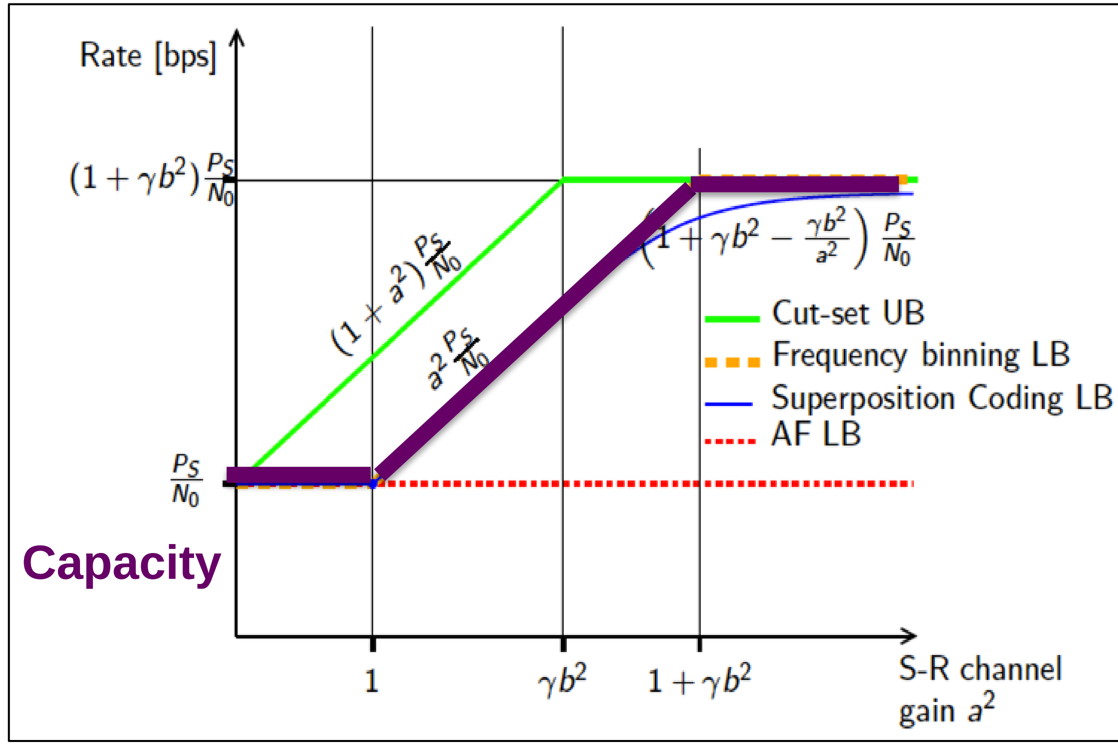


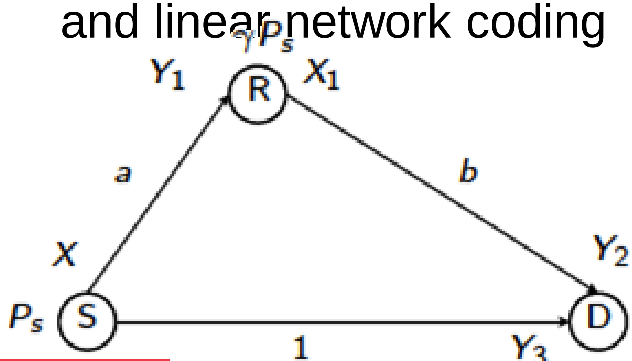
- Multiple access
  - No interference, FDMA
  - Both sources achieve same rate as in the absence of the other user

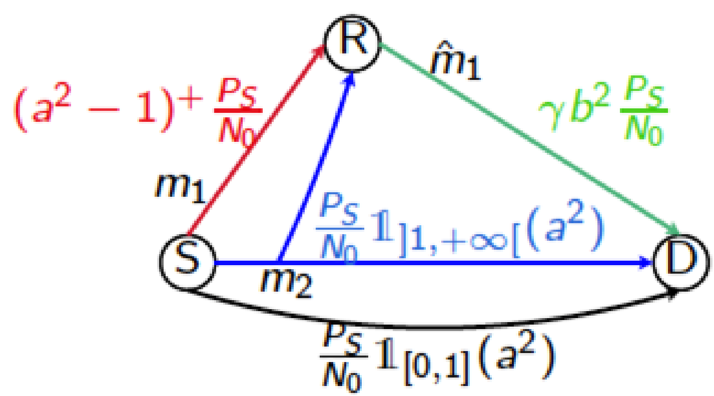


Perils of Virtual MIMO

- SIMO bound is loose in low SNR
- Any given quantization level is insufficient to transmit an uncoded, still noisy version of the data
- Example: in relay network SIMO bound is loose
- At low SNR, network becomes equivalent to a set of edges and hyperedges, with PHY-layer decoding and linear network coding







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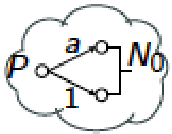
What Min-cut?

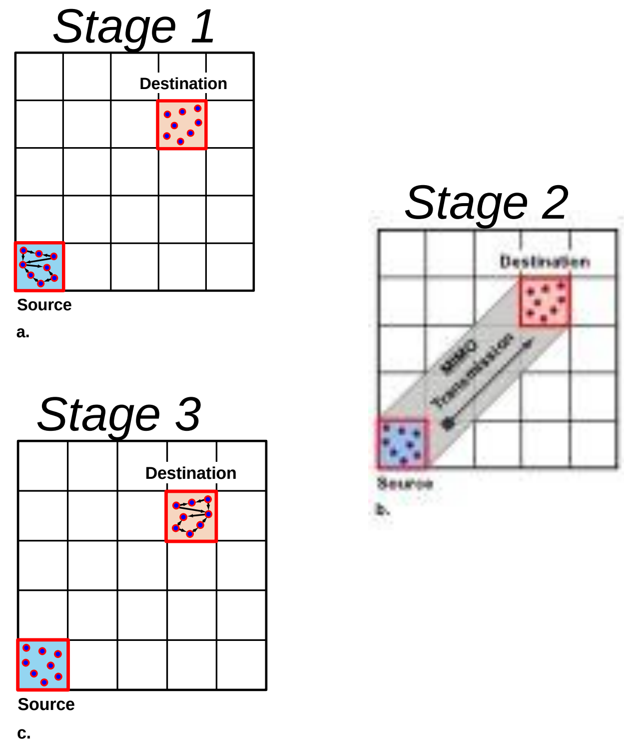
- Open question: Can the gap to the cut-set upper-bound be closed?
- An ∞ capacity on the link R-D would be sufficient to achieve the cut like in SIMO [Kramer et al '05, '06]



- In the limit of a large bandwidth, if the relay cannot decode, large noise power and finite R − D link capacity render the relay contribution useless- **Cannot reach the SIMO cut-set upper-bound [Fawaz, M. '11]**
- Proof relies on rate-distortion theory and equivalence theory
- For physically degraded BC or when the source uses the channel as such, peaky binning relaying is optimal [Fawaz, M. '11]
- Optimum is then selective decode and forward – network is indeed a set of hyperedges

Perils of Virtual MIMO

- In *dense* networks, at high SNRs, SINRs are low.
  - Spectrum segmentation to avoid interference, requires infinite bandwidth
  - Therefore, does not scale
- In *extended* networks, SIMO bound does not hold in low SNR

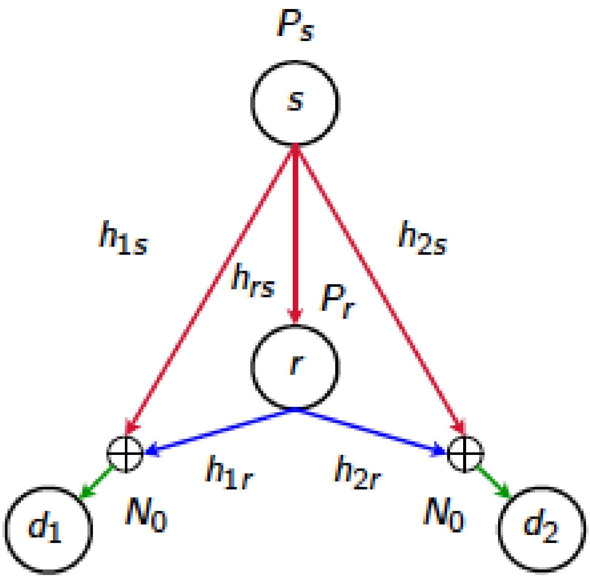


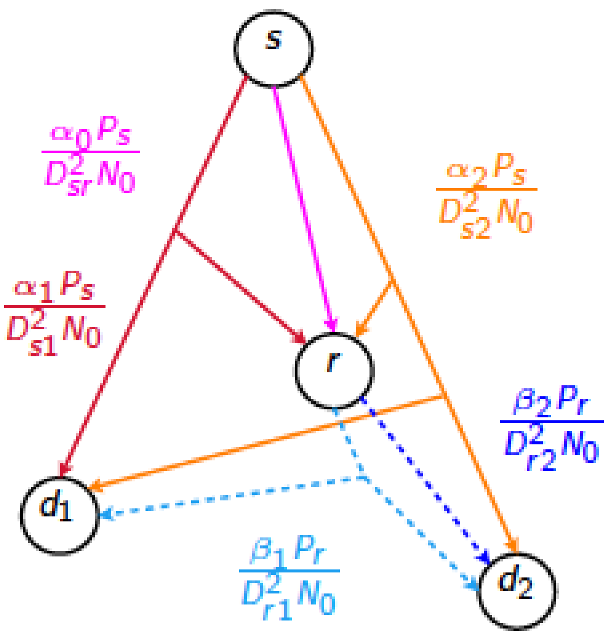
[Zeger, M. ‘13]

Assumption	Dense	Extended	Stage Required
Quantization & SNR Scale Invariance	☑		Stage 3
Spectrum Segmentation		☑	Stage 1 & 3

Practical Implications

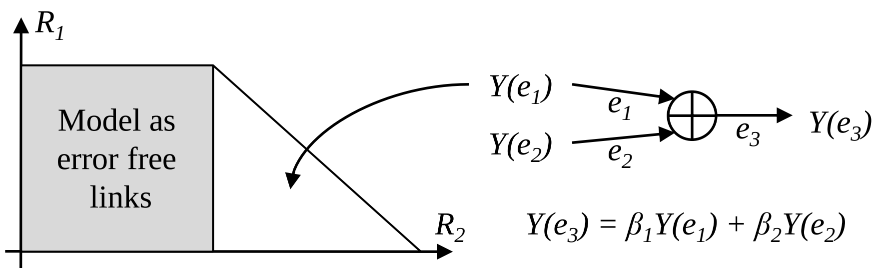
- Achievable hypergraph model: Superposition coding, FDMA.
- Multicommodity flow optimization => Linear program for simple costs (network power, linear cost functions etc.).
- Separable dual => decentralized solutions.
- Hypergraph model can be used to design wireless networks by placing relays [Thakur, M. ‘10, Thakur, Fawaz, M. ’11, ‘12]
- Allows interesting geometric programming with results close to optimum





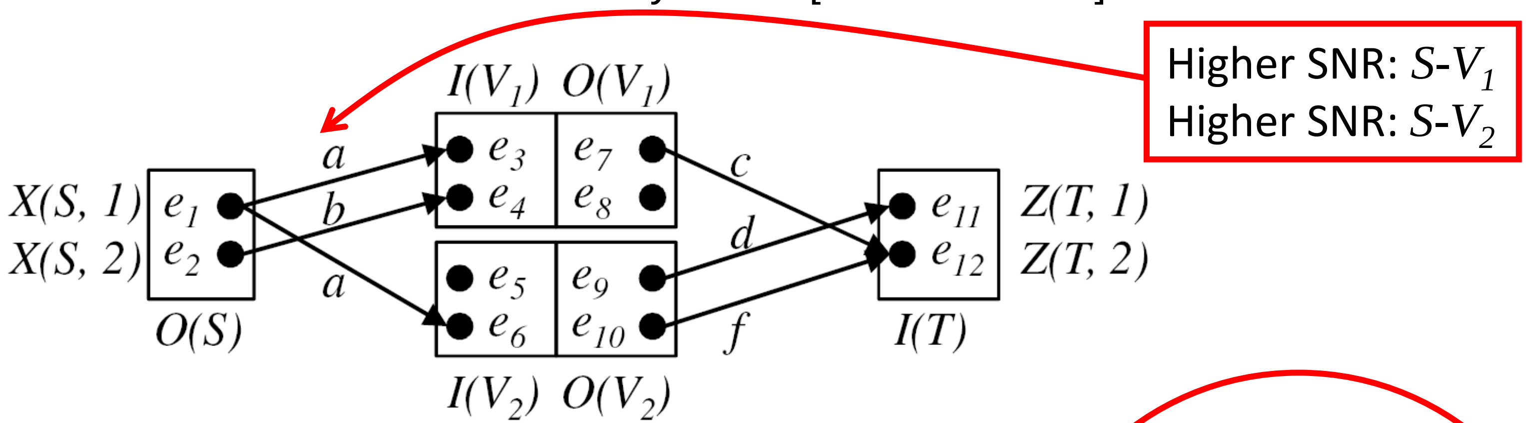
High SNR

- Open problem: capacity & code construction for wireless relay networks
  - Channel noise
  - Interference
- [Avestimehr et al. '07]"Deterministic model" (ADT model)
  - Interference
  - Does not take into account channel noise
  - In essence, high SNR regime of the Cover-Wyner region
  - Separation of network coding and underlying physical channel
  - Loss of 0.5 bits/s/Hz

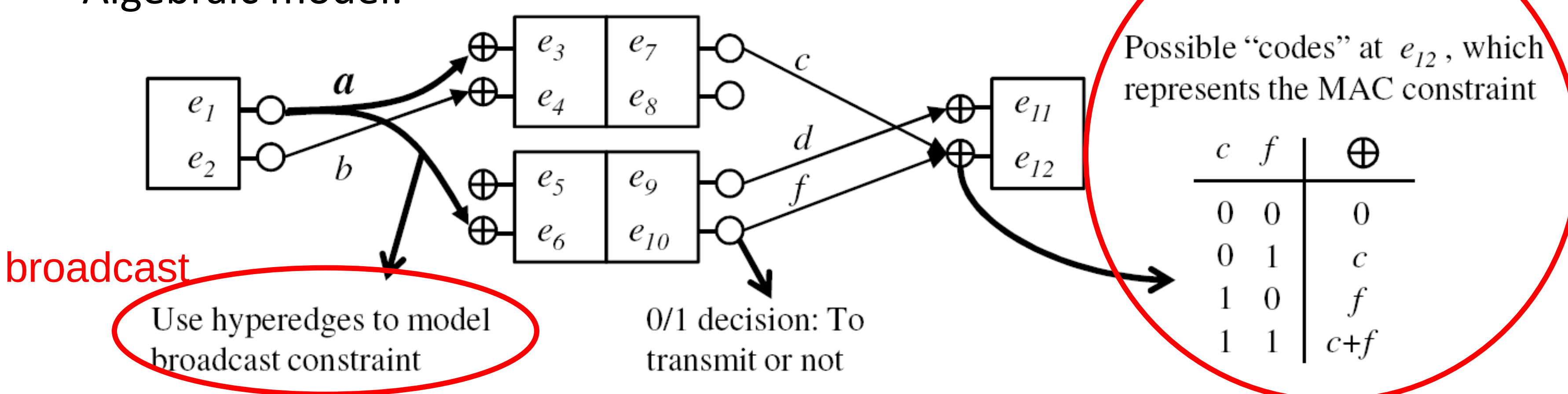


ADT Network Model

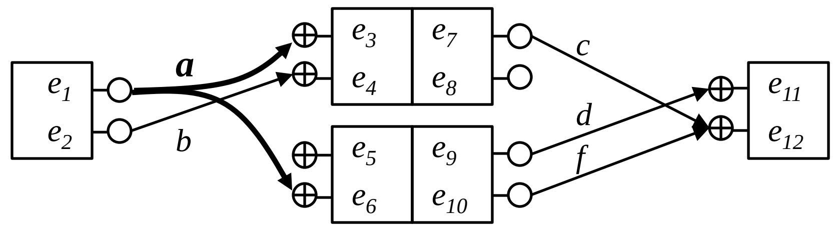
- Original ADT model:
  - Broadcast: multiple edges (bit pipes) from the same node
  - Interference: additive MAC over binary field – [Effros et al ‘04]



- Algebraic model:



System Matrix



- Linear operations
  - Coding at the nodes V: $\beta(e_j, e_{j'})$
    - F represents physical structure of the ADT network
    - F^k: non-zero entry = path of length k between nodes exists
    - $(I-F)^{-1} = I + F + F^2 + F^3 + \ldots$: connectivity of the network (impulse response of the network)

$$F = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_{(e_3,e_7)} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_{(e_4,e_7)} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_{(e_6,e_9)} & \beta_{(e_6,e_{10})} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Broadcast constraint (hyperedge)

MAC constraint (addition)

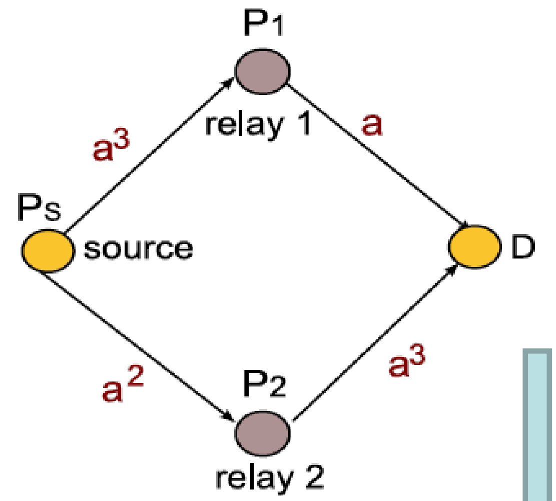
Internal operations (network code)

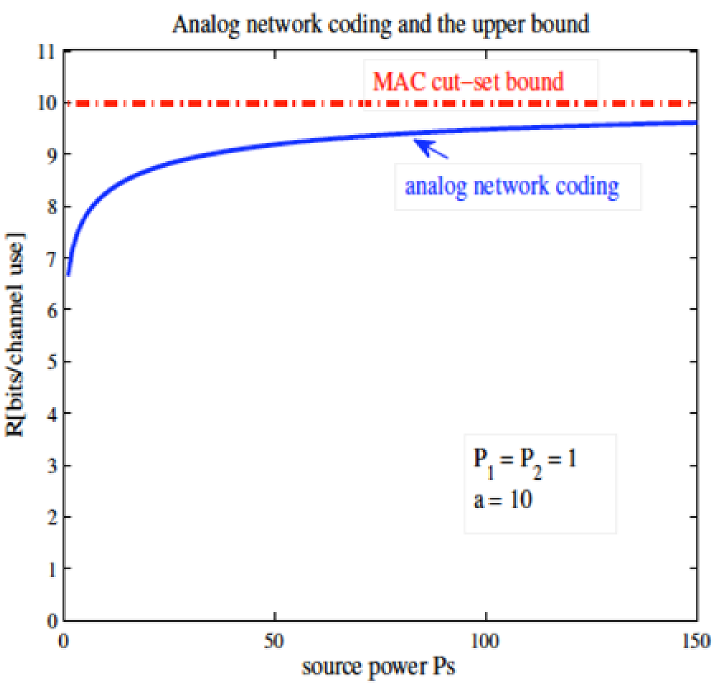
Algebraic Connection

- [Avestimehr et al. ’07] requires optimization over a large set of matrices
- [Kim and M. ‘10] ADT network can be expressed with Algebraic Network Coding Formulation [Koetter and M. ’01, ‘02, ’03]:
  - Model broadcast constraint with **hyper-edge**
  - Rank of single **system matrix** M maps to **physical** min-cut of hypergraph
- Prove an algebraic definition of **min-cut** = rank(M)
- Prove Min-cut Max-flow for unicast/multicast holds
- Extend optimality of linear operations to **non-multicast** sessions
- Show that **random linear network coding** achieves capacity
- Incorporate **failures, random erasures** [Lun et al ‘08, Dana et al ‘05] and **delay** (allows cycles within the network) [Koetter and M. ‘02, ’03]

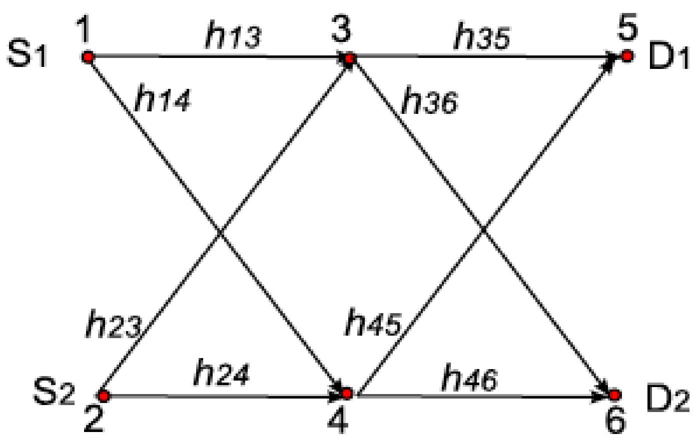
SNR in Networks

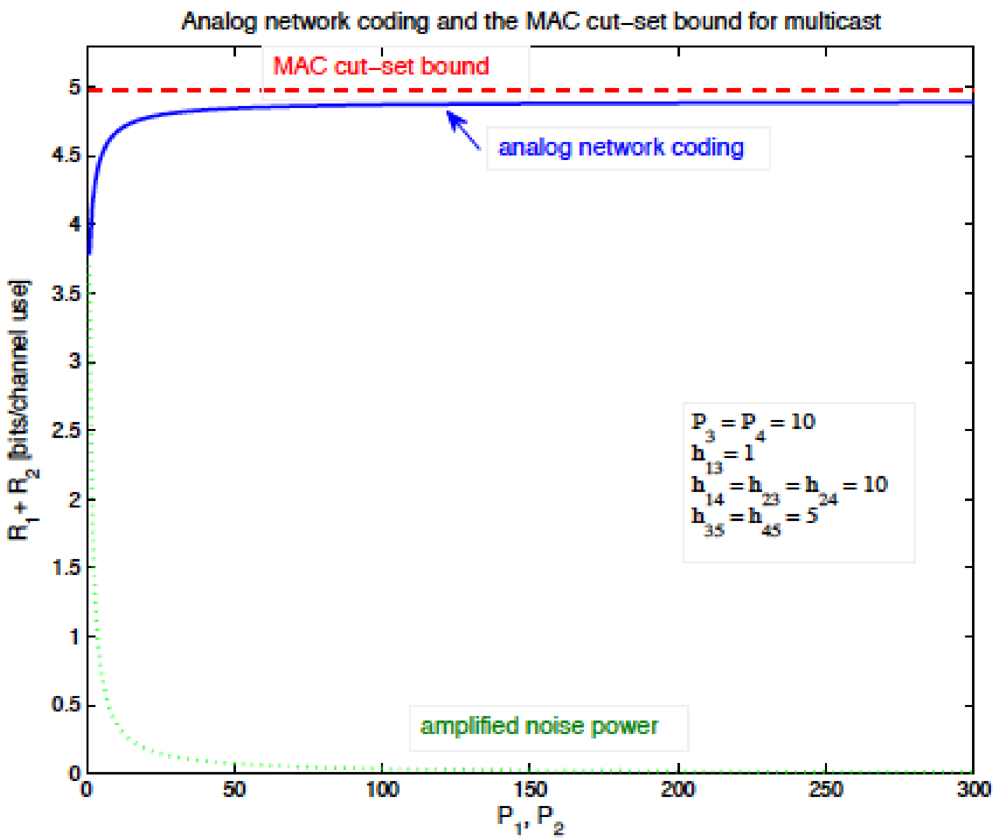
- High SNR in a link
  - Noise $\rightarrow$ 0
  - Large gain
  - Large transmit power
- Consider diamond network [Schein, Gallager' 01]
- Gain:
  - increase *a* [Avestimehr et al '07]
- Large transmit power
  - Amplify-and-forward in the network, ignorant of topology
  - Asymptotically optimal





Analog Network Coding Optimal at High SNR

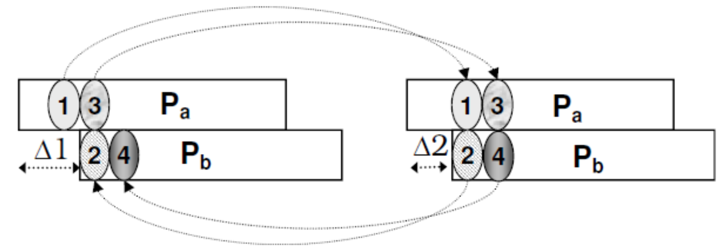




[Maric, Goldsmith, M. '10, '12]

Practical Implications – Zig-Zag Decoding

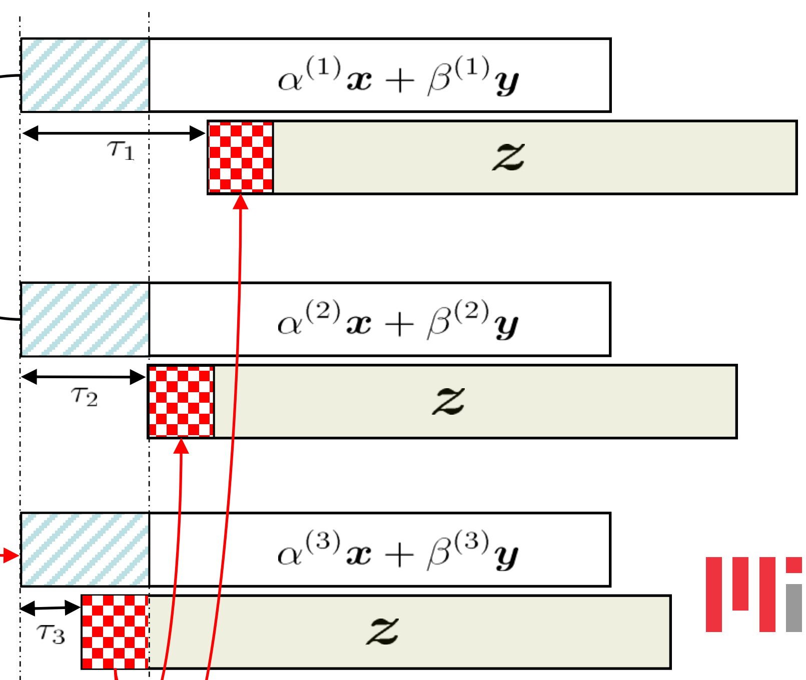
- Successive interference cancellation is a form of analog network coding in high SNR
- Basis for zig-zag [Gollakota and Katabi ‘08]
- Chunk 1 from user A from 1st copy of collided packet can be decoded successfully
  - Subtract from 2nd copy to decoded the Chunk 1 of user B
    - Subtract from 1st copy of collided packet to decode Chunk 2 from user A
      - Subtract from 2nd copy of collided packet to decode Chunk 2 from user B



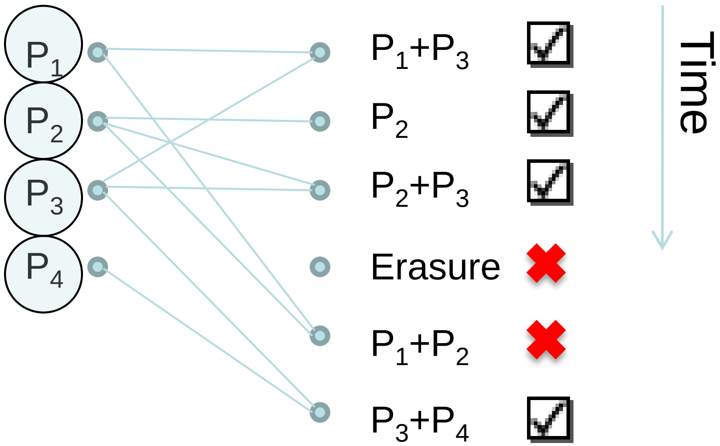
- Can be extended to coded packets

Decode the first chunk of x and y using two interference free portions

Combination of algebraic network coding and analog network coding



Delivery Time - Collision Recovery



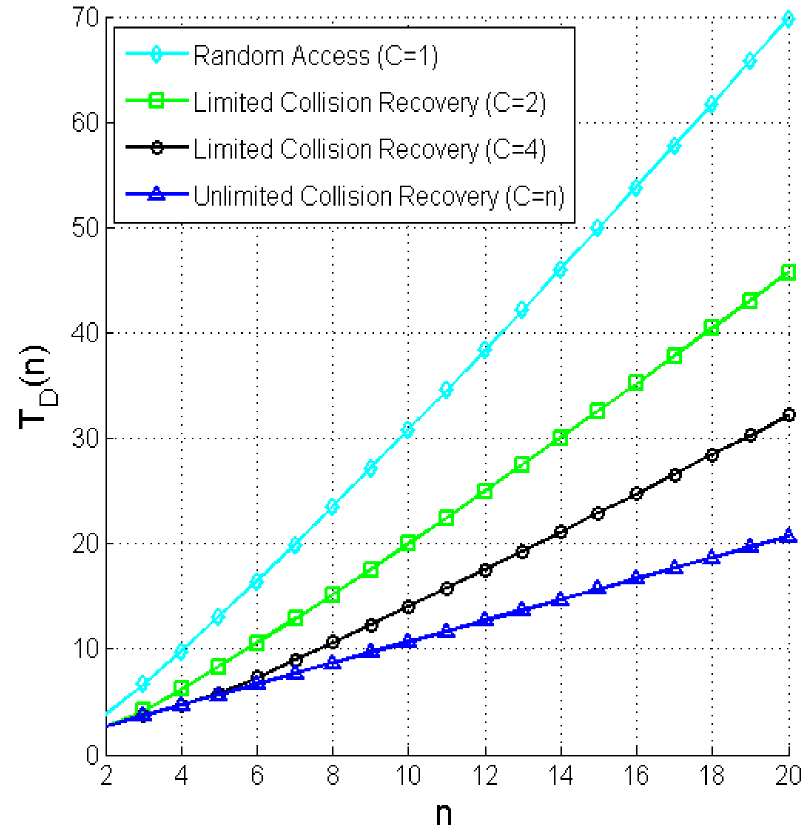
Innovative reception: At least one *undecoded* packet is connected

T_i = Time of reception of i^{th} degree of freedom

$$T_D = T_n = \sum_{i=1}^{n} X_i, \quad \text{where } X_i = T_i - T_{i-1}$$

D_i = Number of decoded packets at time T_{i-1}

$$(X_i | D_i) \sim \text{Geom}\left(\frac{1}{1 - p^{n-D_i}}\right)$$



[ParandehGheibi, Sundararajan, M. '10]

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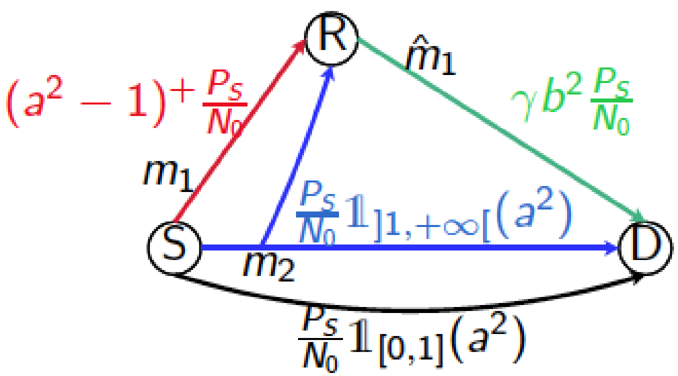
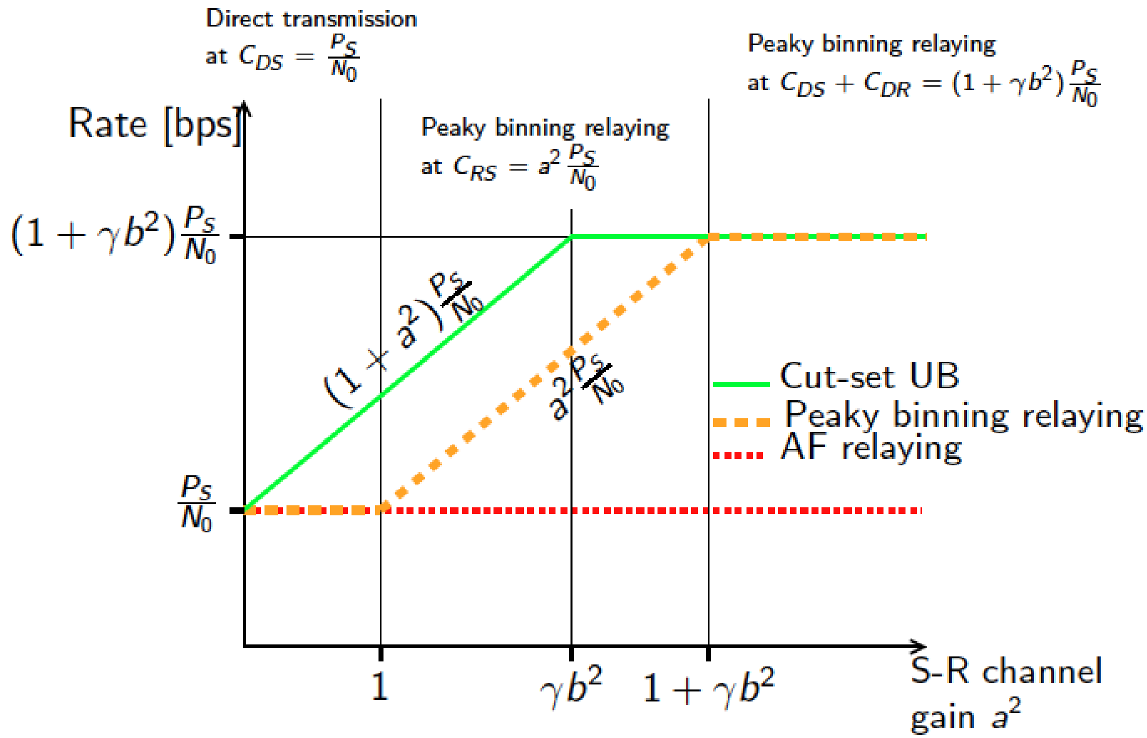
What about Low SNR?

- Consider again hyperedges
- At low SNR, noise is the main issue
- Non-coherence is not bothersome [Fawaz, M. '10]





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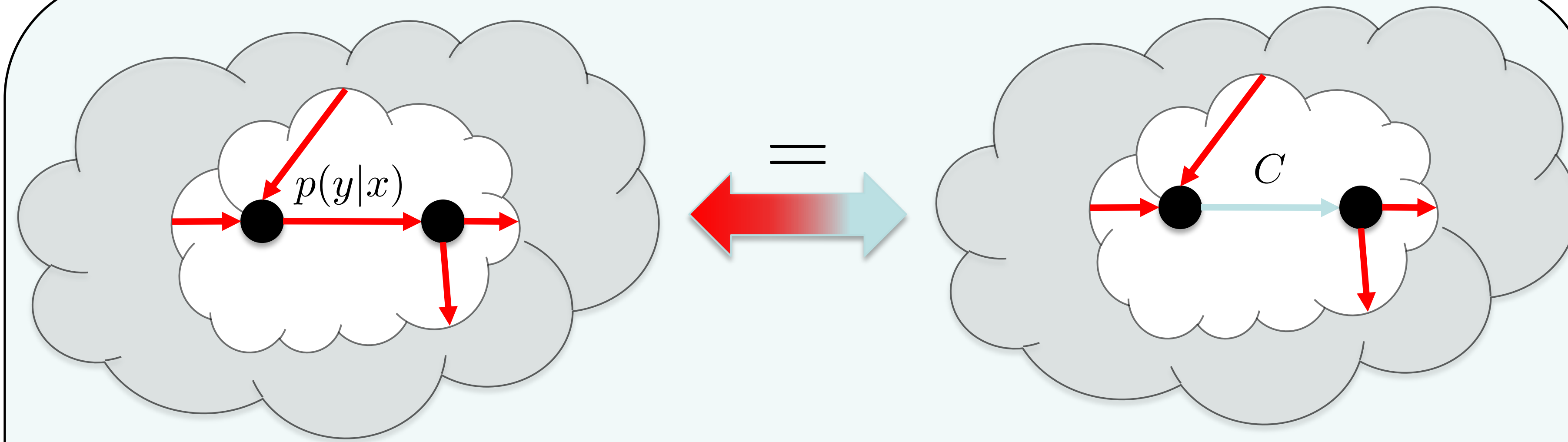
What About Other Regimes?

- The use of hyperedges is important to take into account dependencies
- In general, it is difficult to determine how to proceed (see the difficulties with the relay channel)
- **Equivalence** leads to certain bounds for multiple access and broadcast channels, but these bounds may be loose
- Separates the issue of physical layer coding from that of network coding
  - The new network should be composed by **bit pipes**. This allows the abstraction of the stochastic nature of the network.
  - Instead of bounding the entire network, create bounding **components** for different elements (*e.g.* channels)

Point-to-Point Equivalence

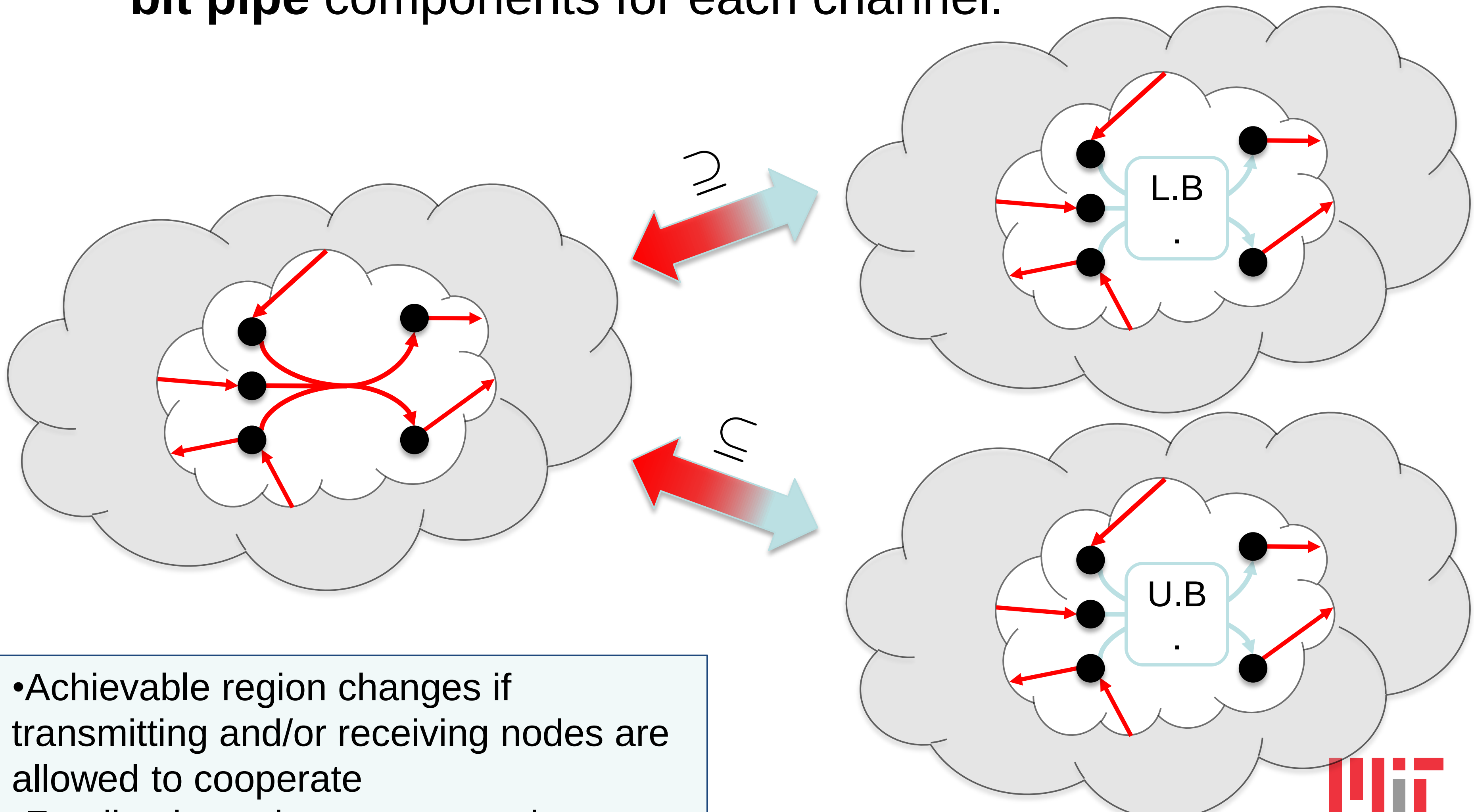
Theorem [Kötter, Effros, M. ‘09, ’11, ‘13]**:** A network composed by discrete memoryless point-to-point links is equivalent to a network where each link is substituted by a noiseless bit pipe with throughput equal to its capacity.



- Consequence: feedback and cooperation cannot increase the achievable rate region.
- How does this extend to networks composed of multi-terminal channels?

Extending to Multi-terminal Channels

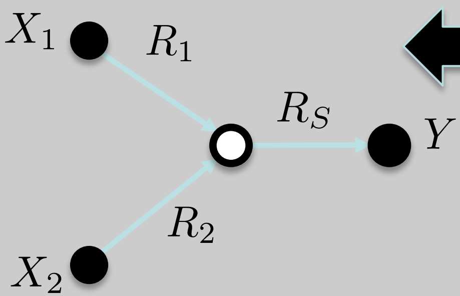
- Key idea: create bounding models by using "equivalent" **bit pipe** components for each channel.



•Achievable region changes if transmitting and/or receiving nodes are allowed to cooperate
•Feedback can increase capacity.

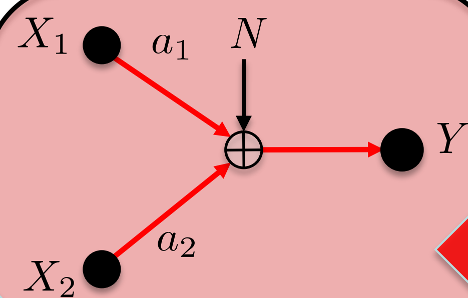
Example: Two User Gaussian MAC

$$\gamma(x) = \frac{1}{2}\log(1+x)$$



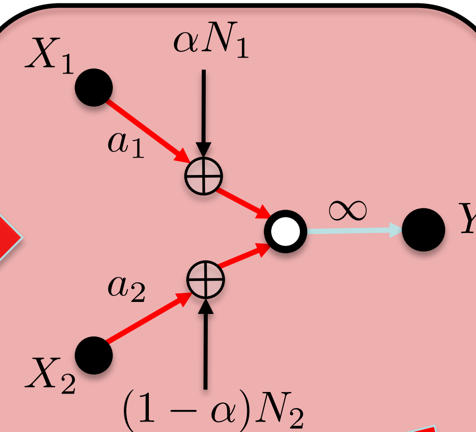
$$R_1 = \gamma(\mathrm{SNR}_1)$$
$$R_2 = \gamma(\mathrm{SNR}_2)$$
$$R_S = \gamma(\mathrm{SNR}_1 + \mathrm{SNR}_2)$$

$\subseteq$



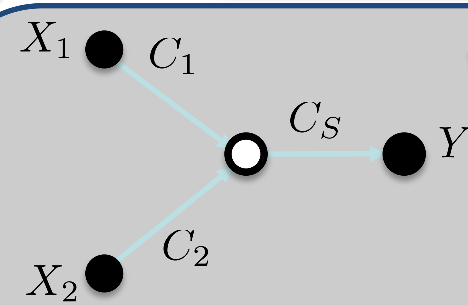
$$Y = a_1 X_1 + a_2 X_2 + N$$
$$\mathbb{E}[X_i^2] \le P_i$$
$$N \sim \mathcal{N}(0, \sigma^2)$$

$\subseteq$



$$N_i \sim N$$

$=$

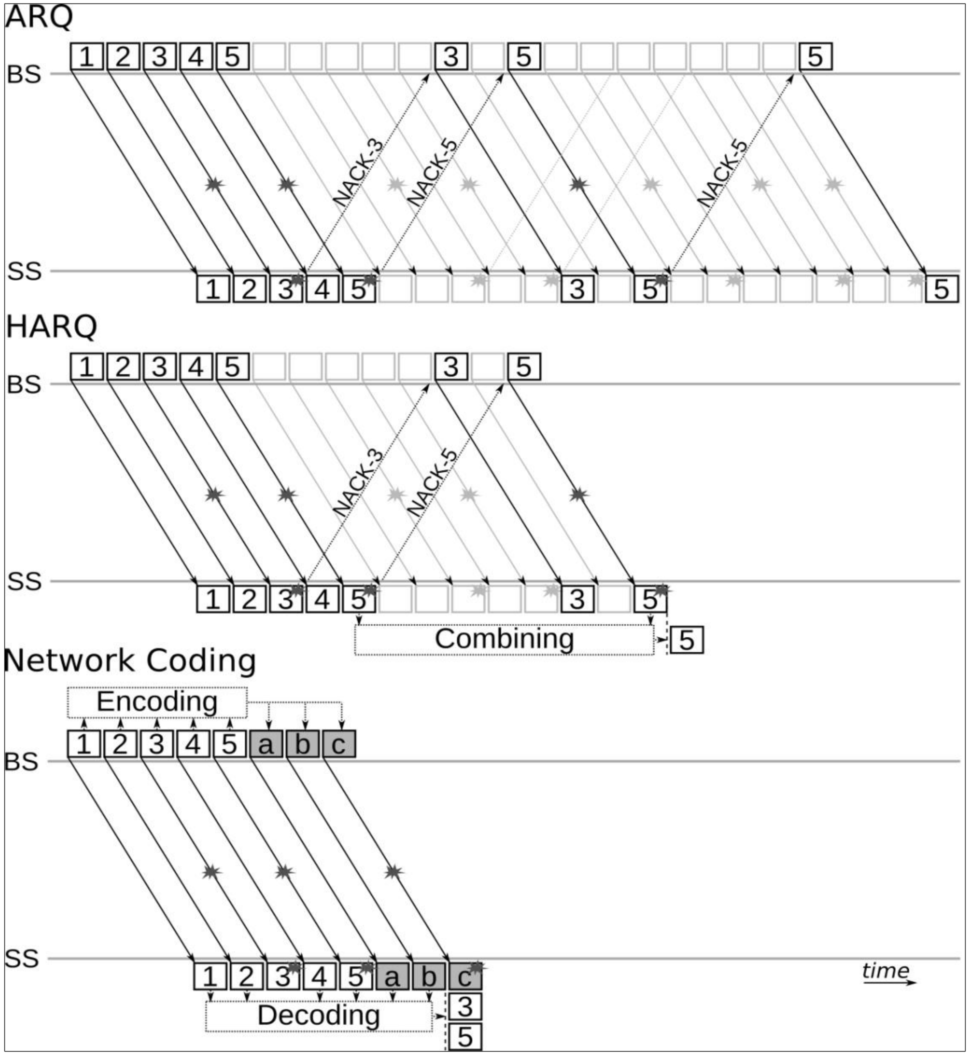


$$C_1 = \gamma(\mathrm{SNR}_1/\alpha^*)$$
$$C_2 = \gamma(\mathrm{SNR}_2/(1-\alpha^*))$$
$$C_S = \gamma((\sqrt{\mathrm{SNR}_1} + \sqrt{\mathrm{SNR}_2})^2)$$
$$\alpha^* = \left(1 + \sqrt{\frac{\mathrm{SNR}_1^{-1}+1}{\mathrm{SNR}_2^{-1}+1}}\right)^{-1}$$

[du Pin Calmon, Effros, M. ‘11]

$$\Delta \le \frac{1}{2}\ \text{bits/s/Hz (smaller at low SNR)}$$

Network Coding vs. ARQ and HARQ

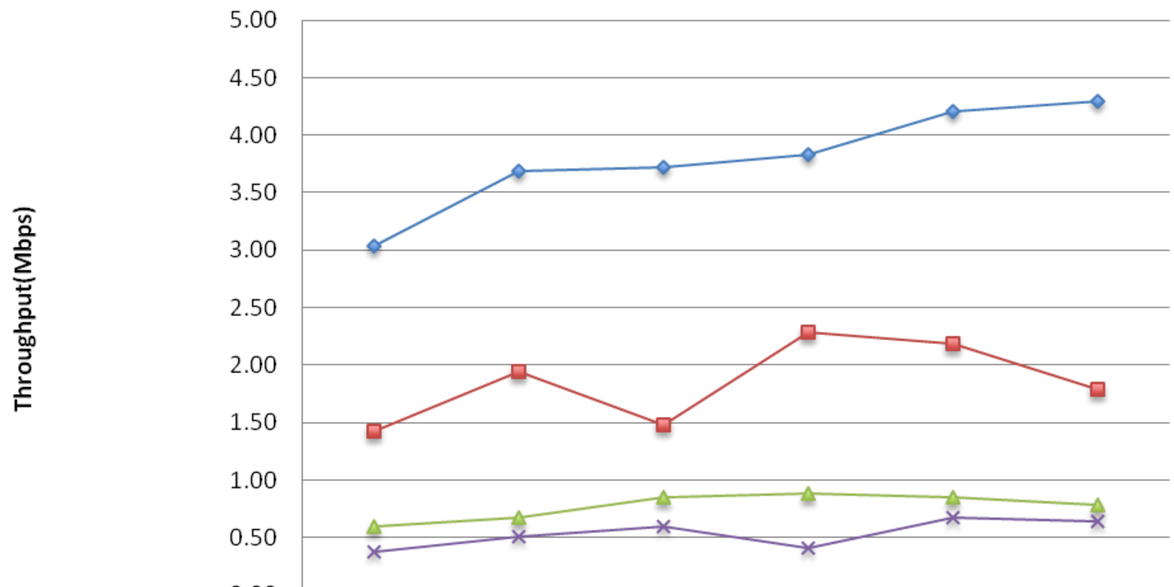


- Scenario: 5-packet block transfer from BS to SS
- Downlink: fixed 40% packet error pattern (every 3rd and 5th packet)
- Uplink: feedback NACKs not subject to loss
- ARQ: repeated transmissions create RTT feedback loops
- HARQ: feedback reduced by combining corrupted packet versions
- Network Coding: a-priori systematic coding with added redundancy of 3/5
- Clear **delay, throughput, and energy gains**
  - No feedback loop
  - Redundancy cost amortized over block

[Teeratapittayanon et al '12]

HARQ and ARQ tests in WiMAX

Global Environment for Network Innovations (GENI) → Indoor experiment at BBN



	70Bytes	140Bytes	280Bytes	360Bytes	720Bytes	1440Bytes
HARQ OFF ARQ OFF	3.03	3.69	3.72	3.84	4.21	4.29
HARQ OFF ARQ ON	1.42	1.94	1.48	2.28	2.19	1.79
HARQ ON ARQ OFF	0.60	0.67	0.84	0.88	0.85	0.79
HARQ ON ARQ ON	0.38	0.51	0.59	0.41	0.67	0.64

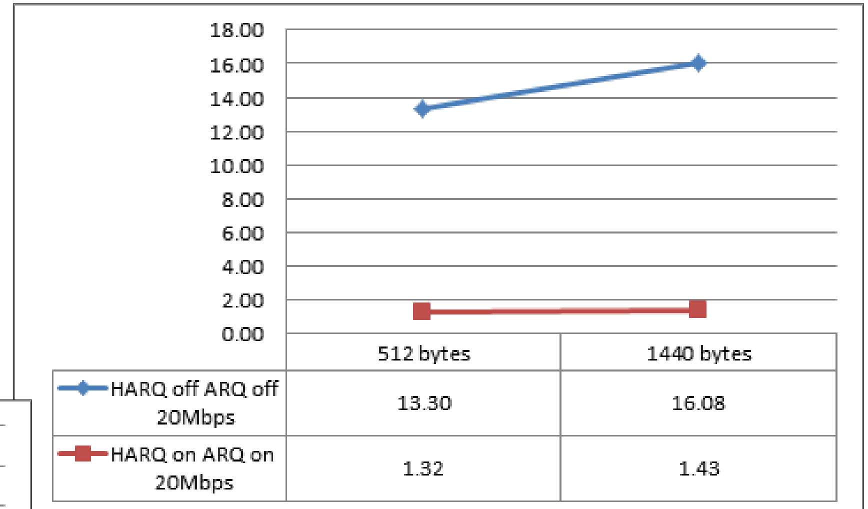
Average downlink throughput (Mbps) for different HARQ/ARQ configurations and packet sizes under 5Mbps offered load

- Turning off HARQ/ARQ increases the available bandwidth in WIMAX
- WiMAX is NOT in any way crucial to our technology – based on availability, through GENI program, of base station
- ARQ and HARQ mechanisms are close between LTE and WiMAX

Consistency

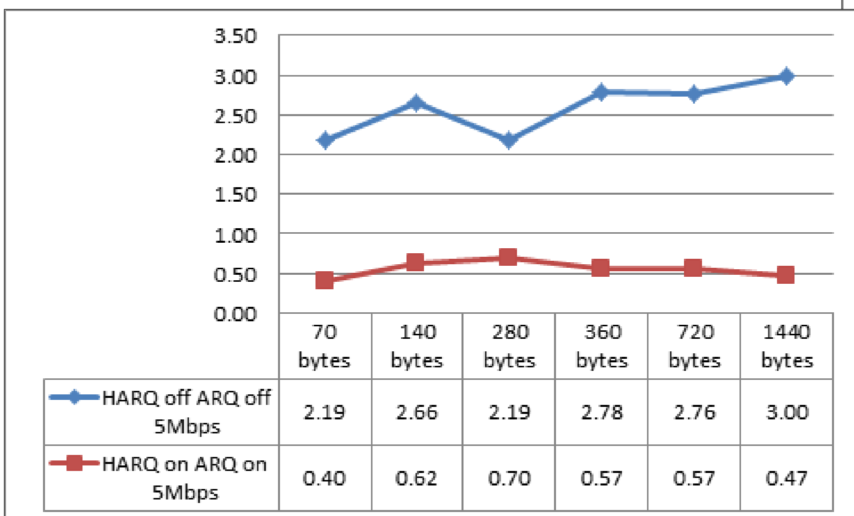
- BBN
- UCLA
- Rutgers

20Mbps



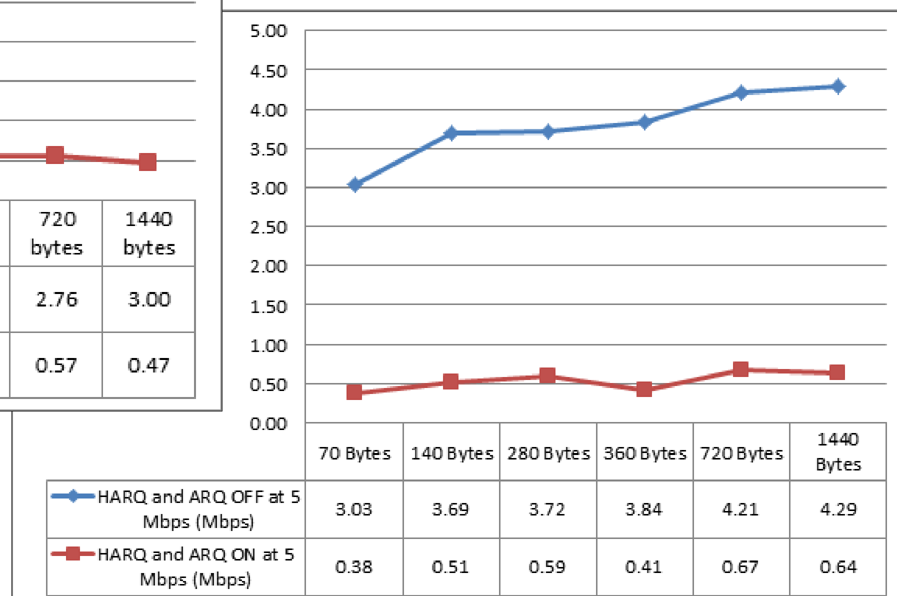
	512 bytes	1440 bytes
HARQ off ARQ off 20Mbps	13.30	16.08
HARQ on ARQ on 20Mbps	1.32	1.43

Opportunity for network coding?



	70 bytes	140 bytes	280 bytes	360 bytes	720 bytes	1440 bytes
HARQ off ARQ off 5Mbps	2.19	2.66	2.19	2.78	2.76	3.00
HARQ on ARQ on 5Mbps	0.40	0.62	0.70	0.57	0.57	0.47

6Mbps (1)



	70 Bytes	140 Bytes	280 Bytes	360 Bytes	720 Bytes	1440 Bytes
HARQ and ARQ OFF at 5 Mbps (Mbps)	3.03	3.69	3.72	3.84	4.21	4.29
HARQ and ARQ ON at 5 Mbps (Mbps)	0.38	0.51	0.59	0.41	0.67	0.64

6Mbps (2)

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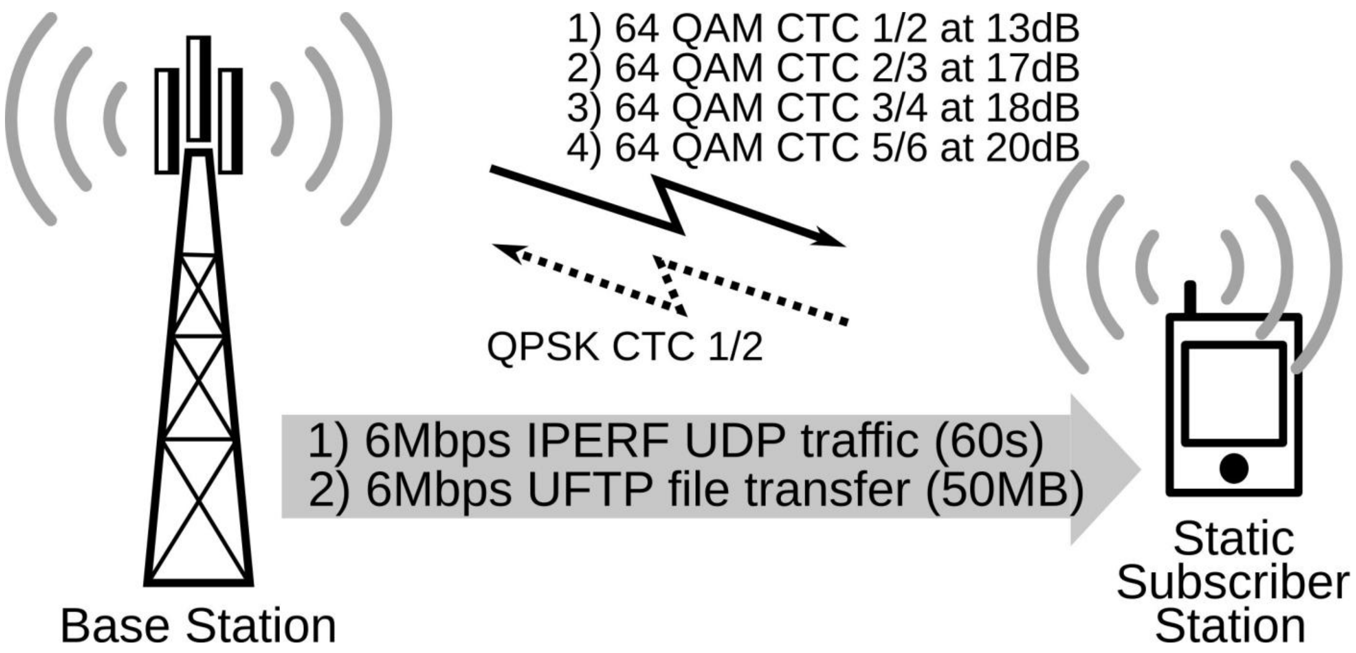
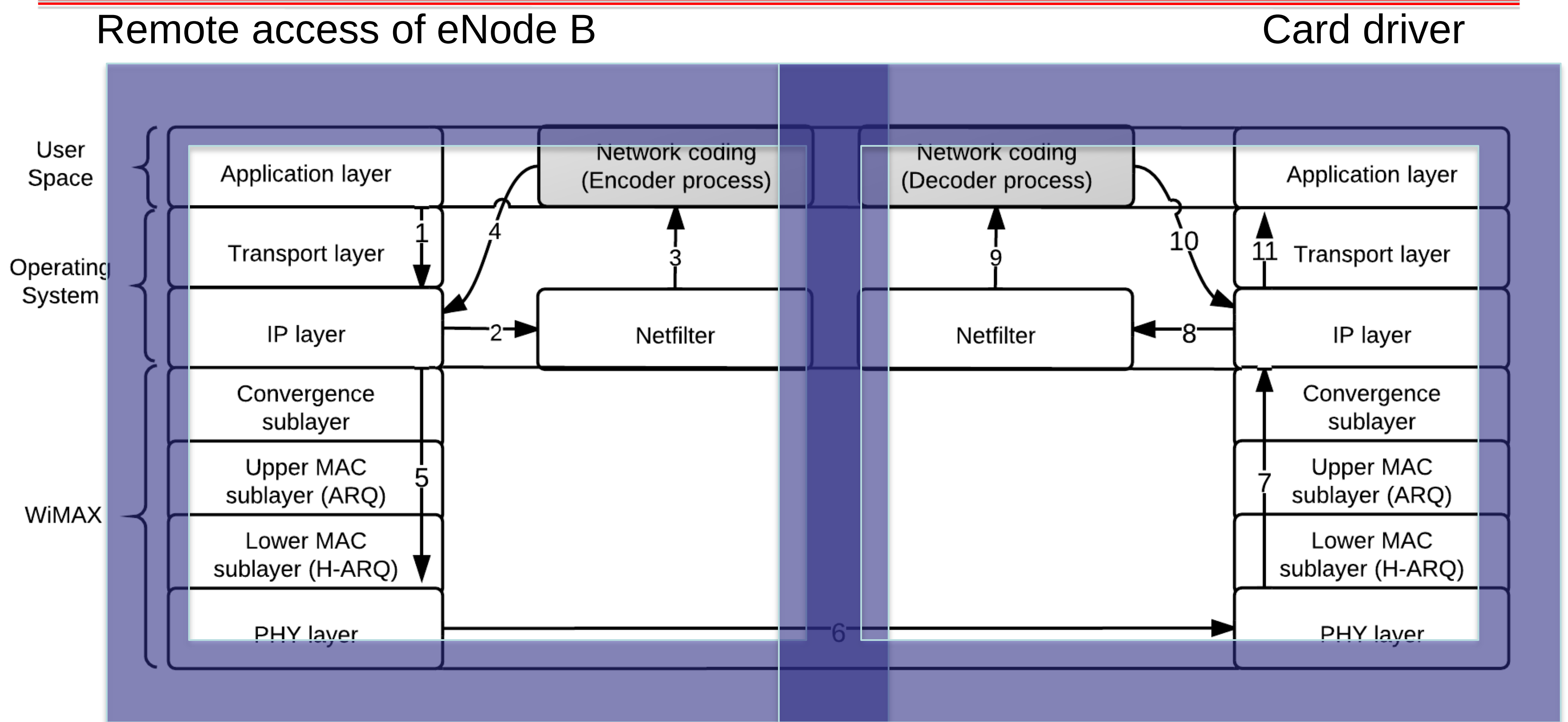
Experimental Setup



- Intra-flow NC modules at the Base Station (BS) and Subscriber Station (SS)
- Toggle ARQ, HARQ, and various NC configurations
- IPERF → application-layer throughput / loss
- UFTP (FTP over UDP) → application-layer file-transfer delay

IP-Based Implementation



- WiMAX MAC inaccessible → IP-based implementation
- Performance measurements at the application layer (IPERF and UFTP)
- IP layer: *Netfilter* used to intercept packets, route them to encoder/decoder, then re-inject them to IP layer
- PDCP does not need to be involved, although that may be quite doable
- Network coding included at the e-Node B before handing it to the MAC, and ARQ and HARQ bypassed at the MAC – does not require a proxy, but can be used if convenient.

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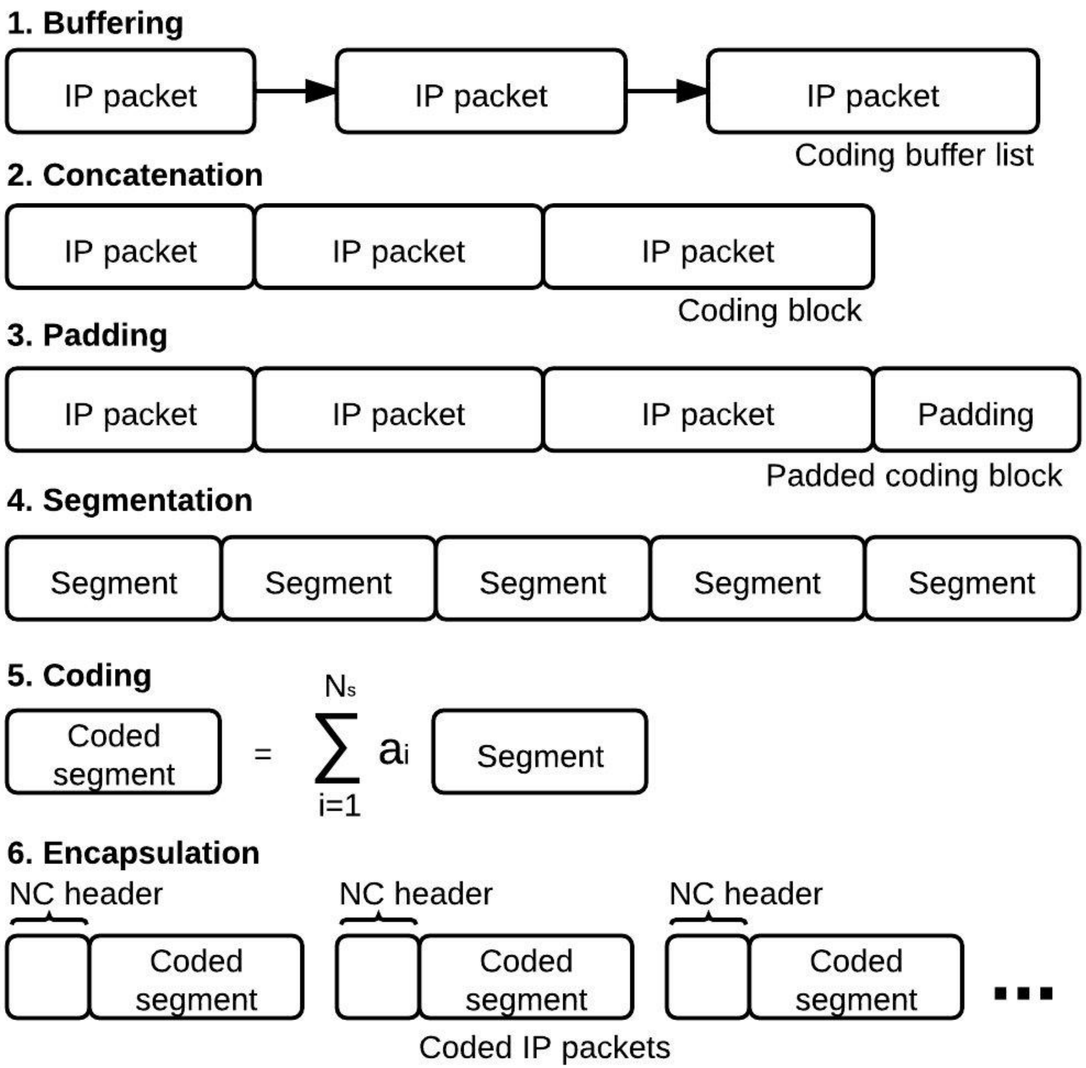
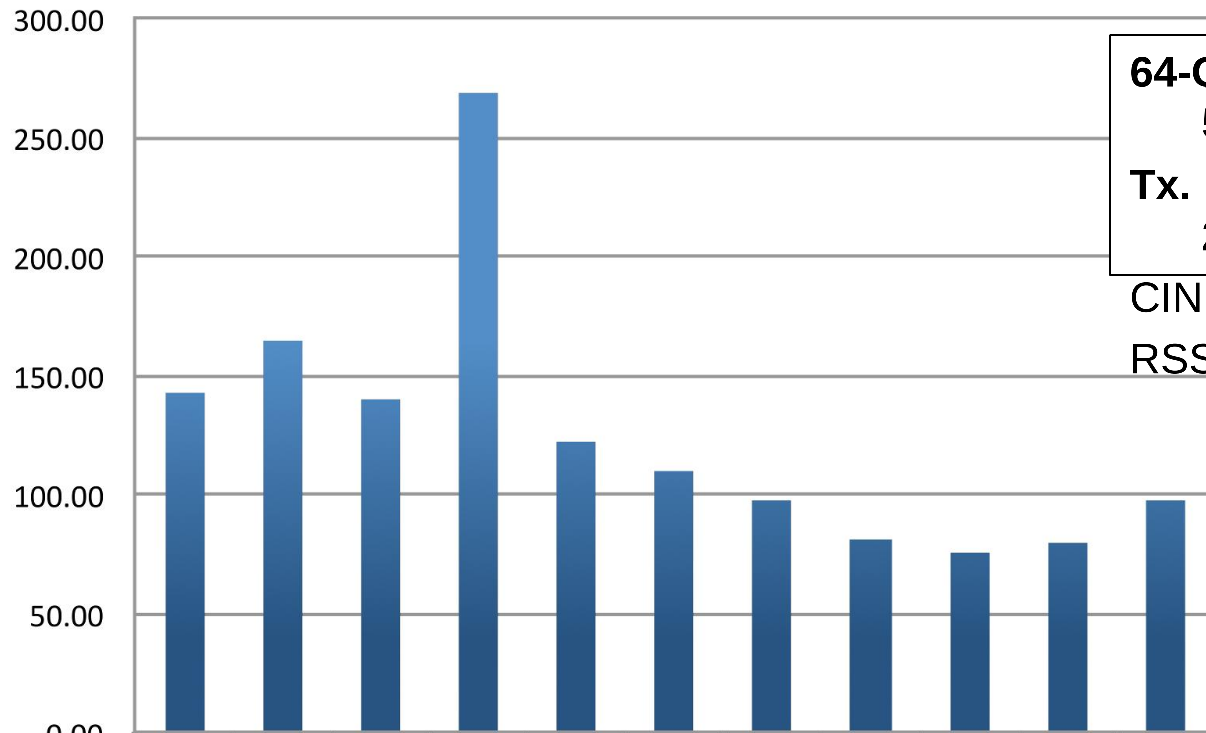
Block-Based Encoding Process



- Concatenate packets up to size- or time-limit
- Pad to minimum block size

- Systematic NC:
  → only redundant packets are coded

Case Study: File Transfer Delay (UFTP)



	Raw	HARQ	HARQ -ARQ	NC-10	NC-15	NC-20	NC-24	NC-30	NC-40	NC-60	NC-120
File Transfer Delay (s)	141.99	164.68	139.34	268.07	122.03	110.02	98.03	81.02	76.06	80.03	98.03

64-QAM CTC 5/6

Tx. Power 20dBm

CINR = 18dB

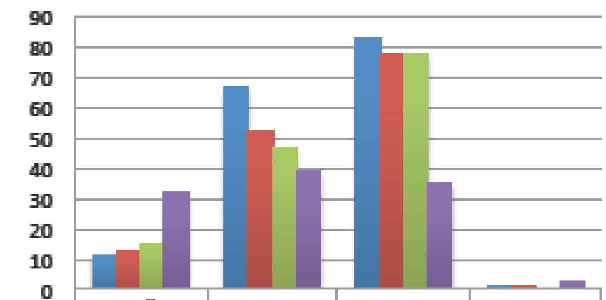
RSSI = -73dBm

- Raw "throughput" unreliable: UFTP runs its own ACK mechanism
- This is not using TCP

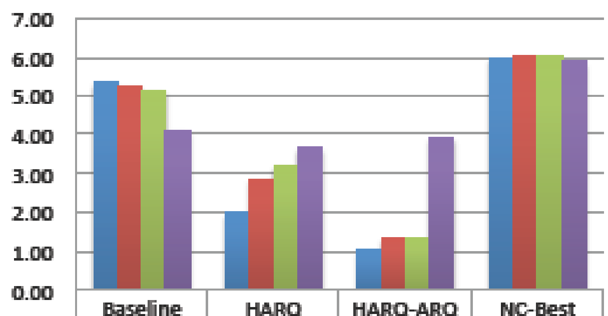
Consistency

- *NC-Best* decreases packet loss **from 11-32% to nearly 0%**
- NC offers up to **5.9x gain in throughput** and **5.5x reduction in file transfer delay**

Loss

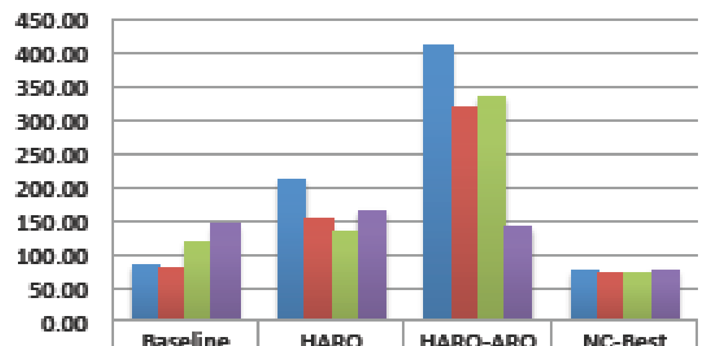


	Baseline	HARQ	HARQ-ARQ	NC-Best
64 QAM 1/2 at 13 dB	11.19	66.5	82.32	0.40
64 QAM 2/3 at 17 dB	12.45	51.63	77.18	0.03
64 QAM 3/4 at 18 dB	14.60	46.37	77.46	0.00
64 QAM 5/6 at 20 dB	31.57	38.60	34.59	2.26



	Baseline	HARQ	HARQ-ARQ	NC-Best
64 QAM 1/2 at 13 dB	5.33	1.97	1.02	5.98
64 QAM 2/3 at 17 dB	5.26	2.86	1.33	5.99
64 QAM 3/4 at 18 dB	5.13	3.18	1.32	5.99
64 QAM 5/6 at 20 dB	4.11	3.66	3.93	5.87

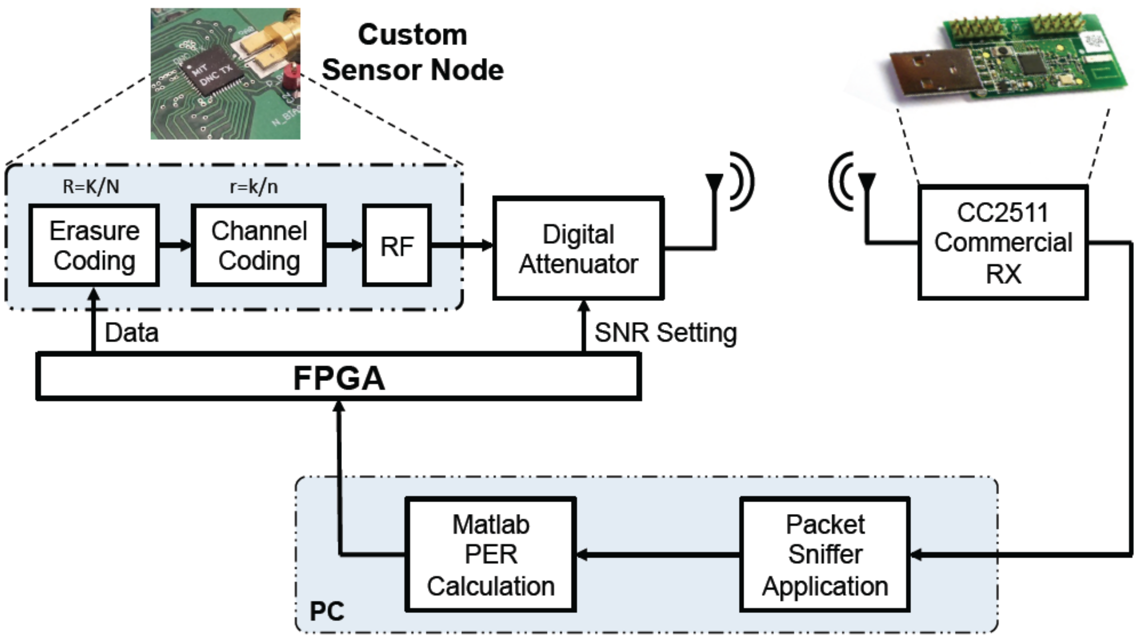
Throughput



	Baseline	HARQ	HARQ-ARQ	NC-Best
64 QAM 1/2 at 13 dB	83.75	210.18	408.37	74.01
64 QAM 2/3 at 17 dB	78.88	150.03	315.82	70.03
64 QAM 3/4 at 18 dB	116.84	133.63	333.71	71.02
64 QAM 5/6 at 20 dB	141.99	164.68	139.34	76.06

File transfer delay

Coding in Sensor nodes



[Angelopoulos, Paidimarri, Chandrakasan, M. ‘13]

Matlab program on a PC through an FPGA.

Generic commercial transceiver (Texas Instruments)

Transmission data rate of 500 kbps

FSK Modulation

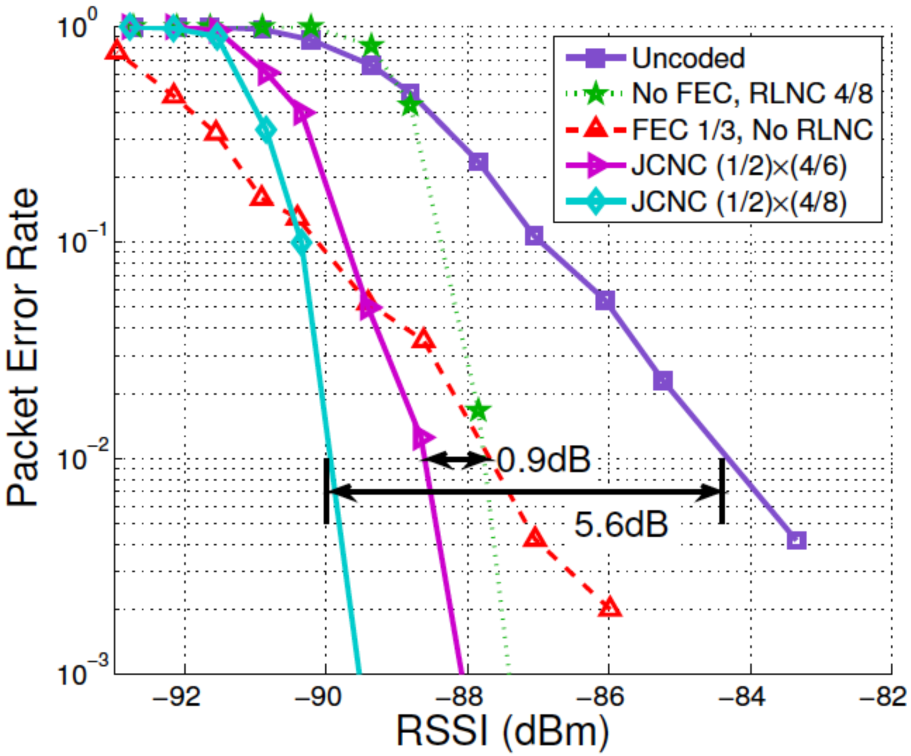
Data transmission and coherent demodulation at receiver

Hard Viterbi decoding and an interleaver of 4 bytes

PC-based packet sniffer software transfers the data from the CC2511 over a USB interface

CC2511 chip provides the Received Signal Strength Indicator (RSSI)

The Benefit of Joint Coding



FEC Rate	RLNC Rate	SNR improvement	
		PER= 10^{-1}	PER= 10^{-2}
1	1	-	-
1	4/5	0dB	1.5dB
1	4/6	0.625dB	2.5dB
1	4/8	1.5dB	3.4dB
1/2	1	2.5dB	2.25dB
1/2	4/5	2.25dB	4dB
1/2	4/6	2.75dB	4.25dB
1/2	4/8	3.5dB	5.6dB

Conclusions

- Joint physical layer and network coding seems in many cases to be limited:
  - Low SNR:
    - Discard noise rather than propagate it
    - Practical implication: simple hypergraph model approximation
  - High SNR:
    - Benefit from performing analog network coding with respect to digitized approach
    - Applications: Zig-Zag generalization
  - In general:
    - Benefits seem to be in general limited
- In practice:
  - WiMax:
    - Remove lower layer approaches by higher-layer coding
  - Low power sensor nodes:
    - Both PHY and network coding are beneficial
    - Coordination between the two may not be necessary
- Suggests an approach that is mostly based on separation – empirical study shows promising results

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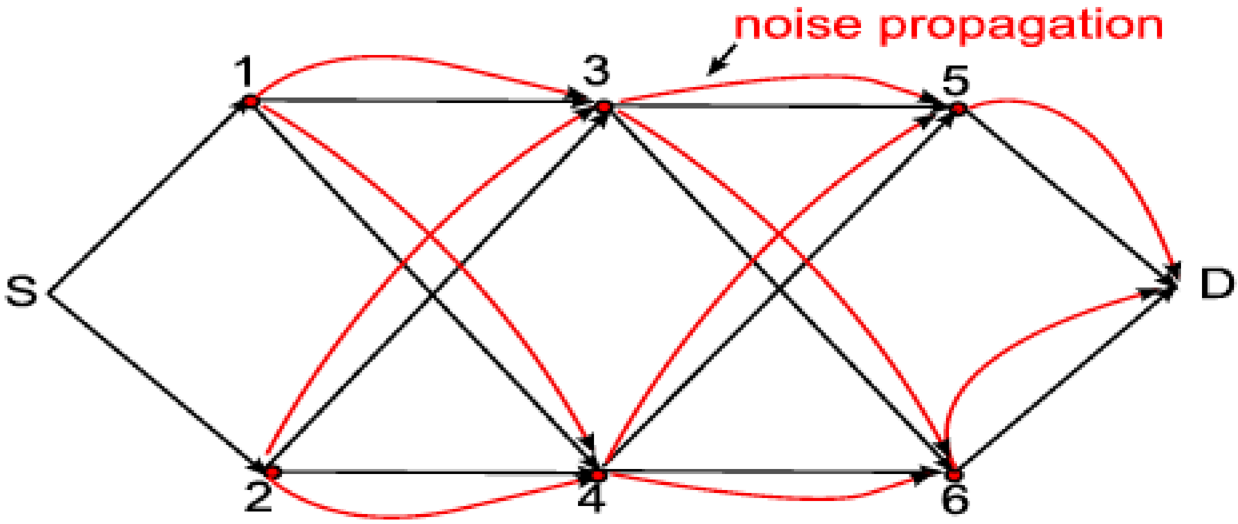
Model

- Denote power at destination $$P_D = \left(\sum_{i \in \mathcal{N}(D)} h_{iD} \sqrt{P_i} \right)^2$$

- MAC cut-set $$C_{\mathrm{MAC}} = \frac{1}{2} \log(1 + P_D)$$

$$y_k = \sum_{j \in \mathcal{N}(k)} h_{jk} x_j + z_k$$



A different view of high SNR

- In a layered relay network under high-SNR conditions:

$$\left(\Big(\sum_{j \in \mathcal{N}(k)} h_{jk} \sqrt{P_j} \Big)^2 \right)^{-1} \le \delta \qquad \text{for each } k \neq D$$

- Analog network coding achieves

Accumulated noise at destination

$$R = \frac{1}{2} \log \left(1 + \frac{1}{(1+\delta)^{L-1}} \frac{P_D}{P_{Z,D} + 1} \right)$$

- At high SNR ANC achieves capacity:

$$P_{Z,D} \le L \delta P_D \qquad R = \frac{1}{2} \log (1 + P_D) - O(\delta)$$

$$\delta \to 0: \qquad P_{Z,D} \to 0$$