IWWSC – Ilmenau
August 30, 2013

Games, Privacy and Distributed Inference for the Smart Grid

Vince Poor
(poor@princeton.edu)

Supported in part by NSF Grant CCF-1016671 and in part by the Marie Curie Outgoing Fellowship Program under Award No. FP7-PEOPLE-IOF-2011-298532.

Overview

Three Topics in Smart Grid:

Overview

Three Topics in Smart Grid:

- *Game Theoretic Methods for Modeling Interactions*

Overview

Three Topics in Smart Grid:

- *Game Theoretic Methods for Modeling Interactions*

- *Privacy-Utility Tradeoffs for Data Sources*

Overview

<u>Three Topics in Smart Grid:</u>

- *Game Theoretic Methods for Modeling Interactions*

- *Privacy-Utility Tradeoffs for Data Sources*

- *Distributed Algorithms for State Estimation*

Game Theoretic Methods for Modeling Interactions

Joint work with Walid Saad, et al.

Introduction & Motivation

- Salient characteristics of smart grid:
  - Heterogeneity: in terms of node types (electric vehicles, smart meters, substations, etc.) with each node having its own objective.
  - Large-scale interactions: spans large geographical areas and could incorporate thousands if not millions of nodes.
  - Stochastic dynamics: time-varying features, in terms of demand, supply, node dynamics (e.g., car mobility), etc.

Introduction & Motivation

- Salient characteristics of smart grid:
  - Heterogeneity: in terms of node types (electric vehicles, smart meters, substations, etc.) with each node having its own objective.
  - Large-scale interactions: spans large geographical areas and could incorporate thousands if not millions of nodes.
  - Stochastic dynamics: time-varying features, in terms of demand, supply, node dynamics (e.g., car mobility), etc.
- Need techniques that capture individual node objectives, large-scale interactions, and dynamics/uncertainty

Introduction & Motivation

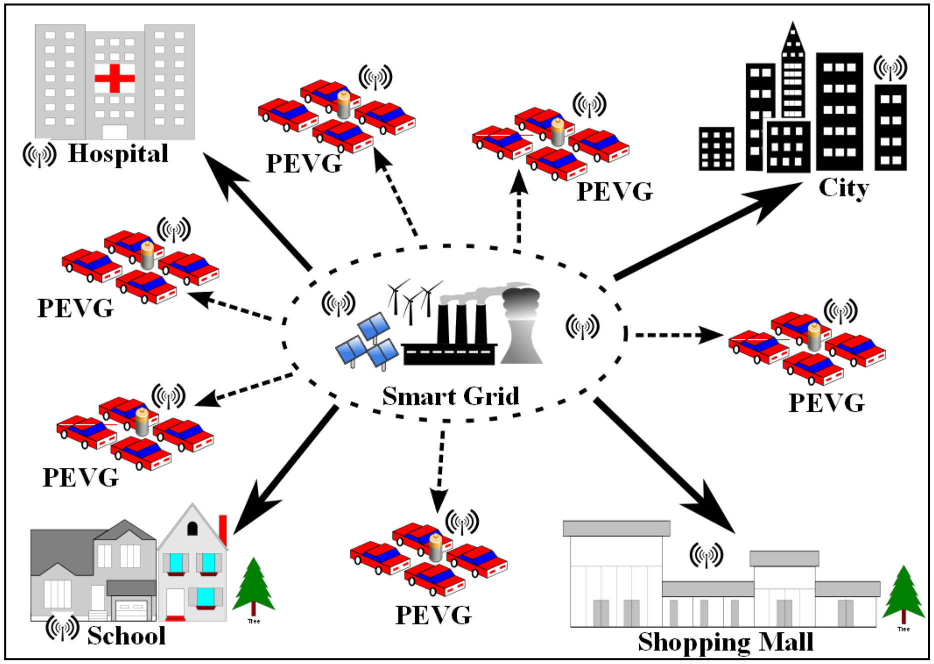
- Salient characteristics of smart grid:
  - Heterogeneity: in terms of node types (electric vehicles, smart meters, substations, etc.) with each node having its own objective.
  - Large-scale interactions: spans large geographical areas and could incorporate thousands if not millions of nodes.
  - Stochastic dynamics: time-varying features, in terms of demand, supply, node dynamics (e.g., car mobility), etc.
- Need techniques that capture individual node objectives, large-scale interactions, and dynamics/uncertainty
- Useful framework - game theory in its two branches:
  - Non-cooperative game theory
  - Cooperative game theory

Introduction & Motivation

- Salient characteristics of smart grid:
  - Heterogeneity: in terms of node types (electric vehicles, smart meters, substations, etc.) with each node having its own objective.
  - Large-scale interactions: spans large geographical areas and could incorporate thousands if not millions of nodes.
  - Stochastic dynamics: time-varying features, in terms of demand, supply, node dynamics (e.g., car mobility), etc.
- Need techniques that capture individual node objectives, large-scale interactions, and dynamics/uncertainty
- Useful framework - game theory in its two branches:
  - Non-cooperative game theory
  - Cooperative game theory
- Illustrate via two examples

Ex. 1: Energy Trading for Plug-In Vehicles

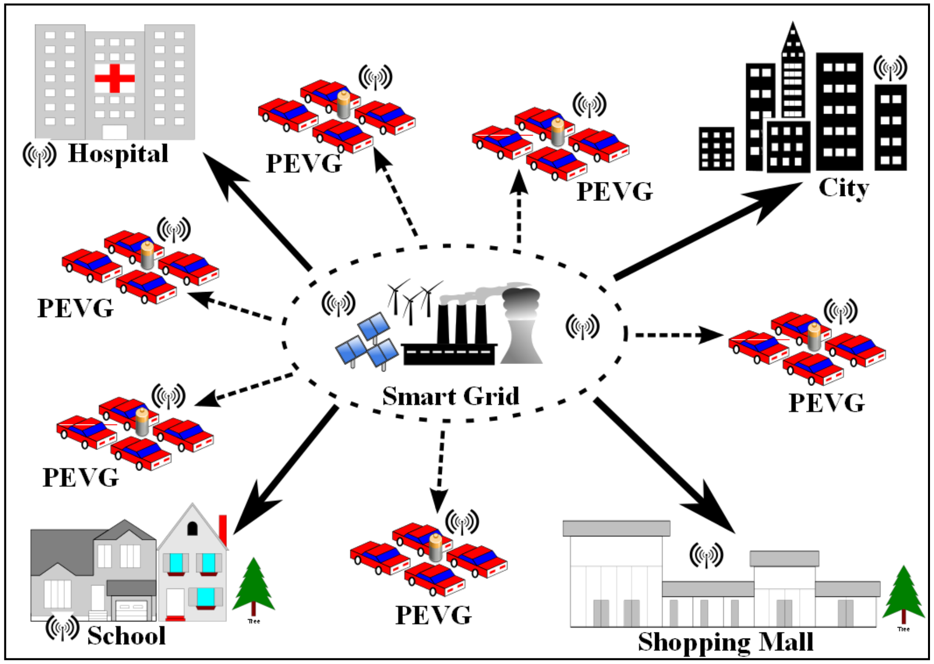
- Groups of plug-in electric vehicles can trade energy with the main grid.



Ex. 1: Energy Trading for Plug-In Vehicles

- Groups of plug-in electric vehicles can trade energy with the main grid.

- Non-cooperative games can model interactions among such groups.

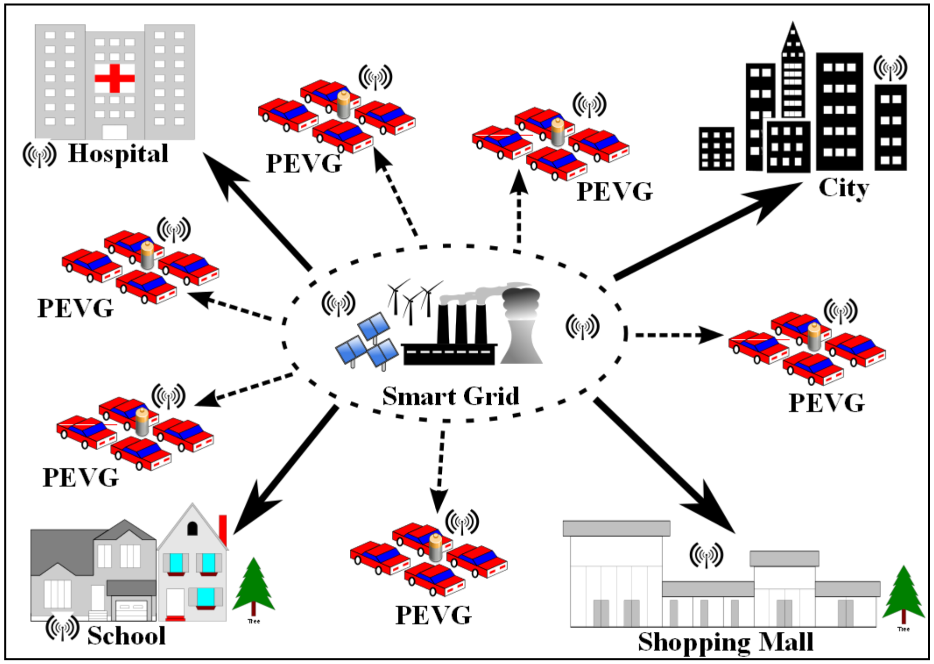


Ex. 1: Energy Trading for Plug-In Vehicles

- Groups of plug-in electric vehicles can trade energy with the main grid.

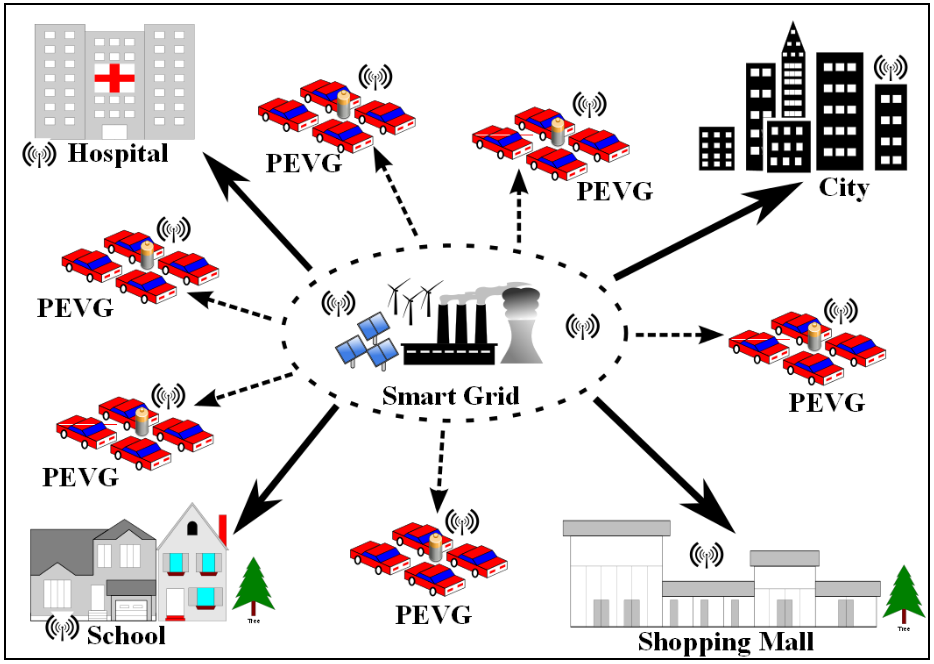
- Non-cooperative games can model interactions among such groups.

- If the grid acts as a single entity, a Stackelberg (leader-follower) game provides a good model.



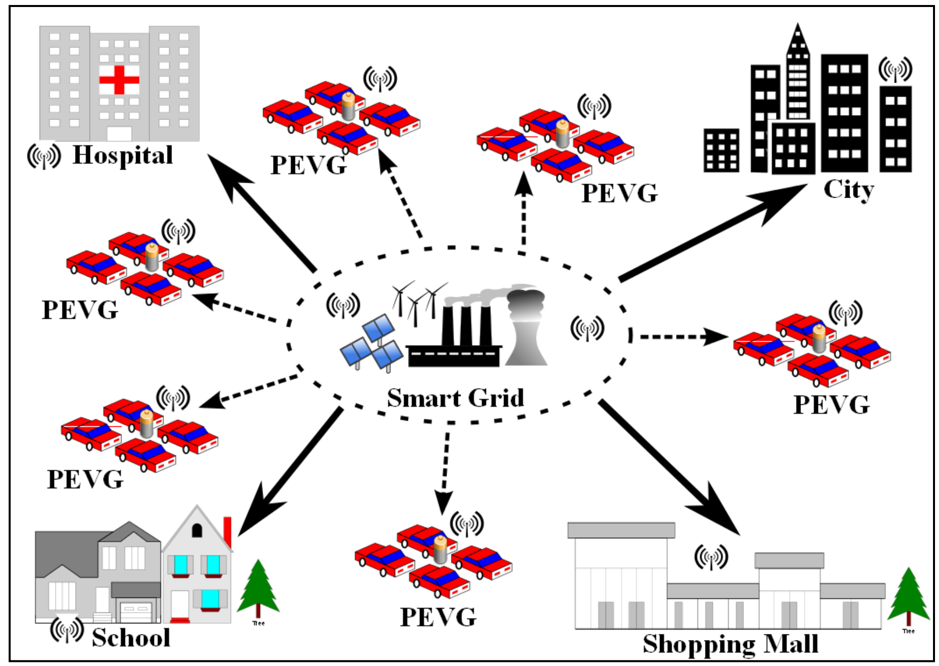
Ex. 1: Energy Trading for Plug-In Vehicles

- Groups of plug-in electric vehicles can trade energy with the main grid.
- Non-cooperative games can model interactions among such groups.
- If the grid acts as a single entity, a Stackelberg (leader-follower) game provides a good model.
- If grid elements act autonomously, a hybrid auction/Nash game can be used.



Ex. 1: Energy Trading for Plug-In Vehicles

- Groups of plug-in electric vehicles can trade energy with the main grid.
- Non-cooperative games can model interactions among such groups.
- If the grid acts as a single entity, a Stackelberg (leader-follower) game provides a good model.
- If grid elements act autonomously, a hybrid auction/Nash game can be used. Consider this first, with the EV groups selling …

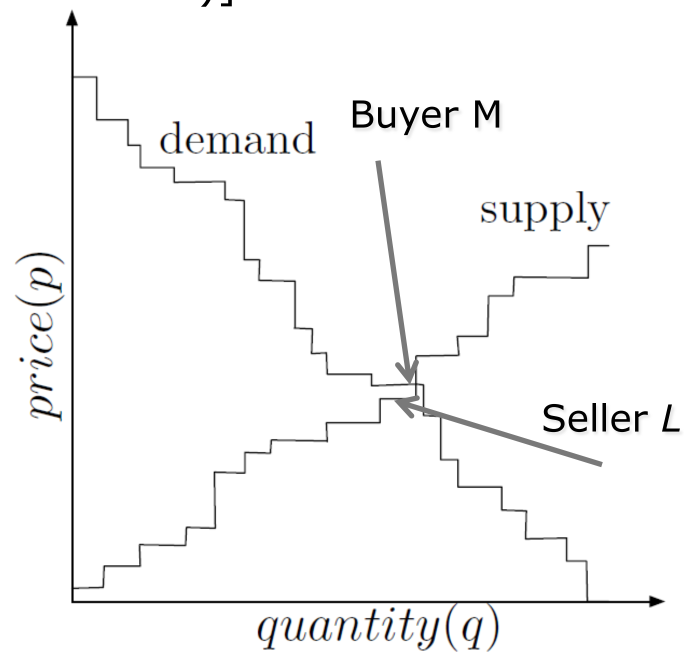


Double Auction Market Model

[w/ Saad, Han, Basar – T-SG (submitted)]

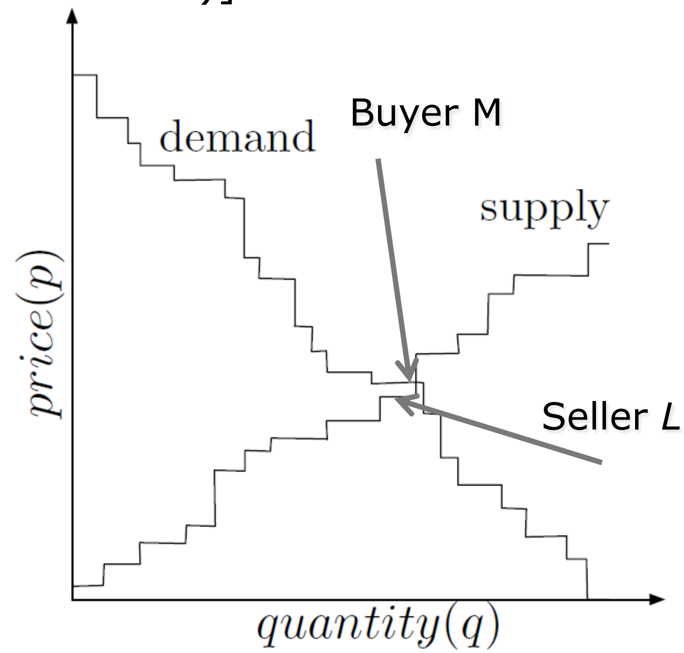
- Double auction:
  - Order buyers by decreasing bids

    and sellers by increasing prices
  - Generate supply-demand curve



Double Auction Market Model

[w/ Saad, Han, Basar – T-SG (submitted)]

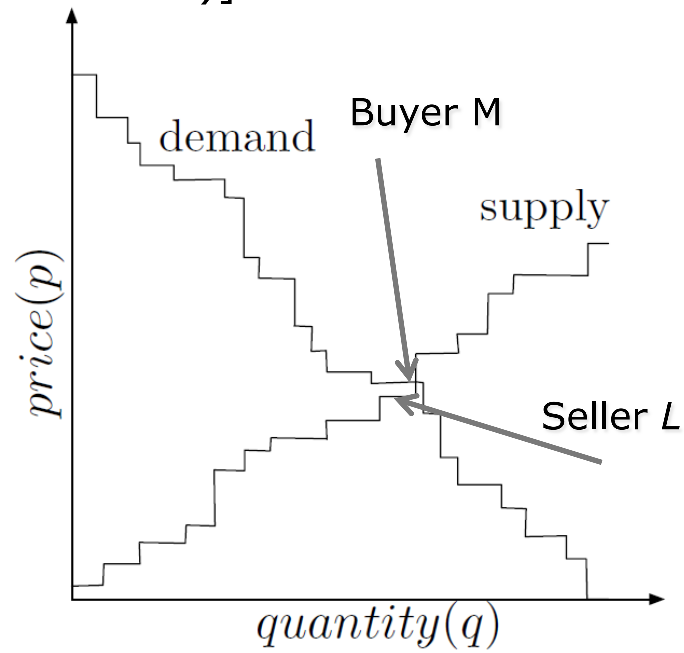


- Double auction:
  - Order buyers by decreasing bids

    and sellers by increasing prices
  - Generate supply-demand curve
  - Intersection: the aggregate demand and supply curve intersect at a point which determines:
    - The number and identity of the sellers and buyers that will trade; assume *L-1* sellers and *M-1* buyers trade

Double Auction Market Model

[w/ Saad, Han, Basar – T-SG (submitted)]



- Double auction:
  - Order buyers by decreasing bids

    and sellers by increasing prices
  - Generate supply-demand curve
  - Intersection: the aggregate demand and supply curve intersect at a point which determines:
    - The number and identity of the sellers and buyers that will trade; assume *L-1* sellers and *M-1* buyers trade
    - The trading price is given by

$$\bar{p}(\boldsymbol{a}) = \frac{s_L + b_M}{2}$$

a is the vector of energy put up for sale, s_L and b_M are the reservation bids of seller *L* and buyer *M*

A Non-Cooperative Game

[w/ Saad, Han, Basar – T-SG (submitted)]

- The strategy of a vehicle group *i* is to choose the maximum amount a_i of energy to sell.

A Non-Cooperative Game

[w/ Saad, Han, Basar – T-SG (submitted)]

- The strategy of a vehicle group *i* is to choose the maximum amount a_i of energy to sell.

- Vehicle group *i* chooses its strategy to maximize its utility:

$$U_i(a_i, \boldsymbol{a}_{-i}) = \underbrace{(\bar{p}(\boldsymbol{a})}_{\text{Trading price (auction outcome)}} - s_i)\underbrace{Q_i(\boldsymbol{a})}_{\text{Quantity sold (auction outcome)}} - \underbrace{\tau_i}_{\text{Pricing factor}} Q_i^2(\boldsymbol{a})$$

A Non-Cooperative Game

[w/ Saad, Han, Basar – T-SG (submitted)]

- The strategy of a vehicle group *i* is to choose the maximum amount a_i of energy to sell.
- Vehicle group *i* chooses its strategy to maximize its utility:

$$U_i(a_i, \boldsymbol{a}_{-i}) = (\bar{p}(\boldsymbol{a}) - s_i)Q_i(\boldsymbol{a}) - \tau_i Q_i^2(\boldsymbol{a})$$

Trading price
(auction outcome)

Quantity sold
(auction outcome)

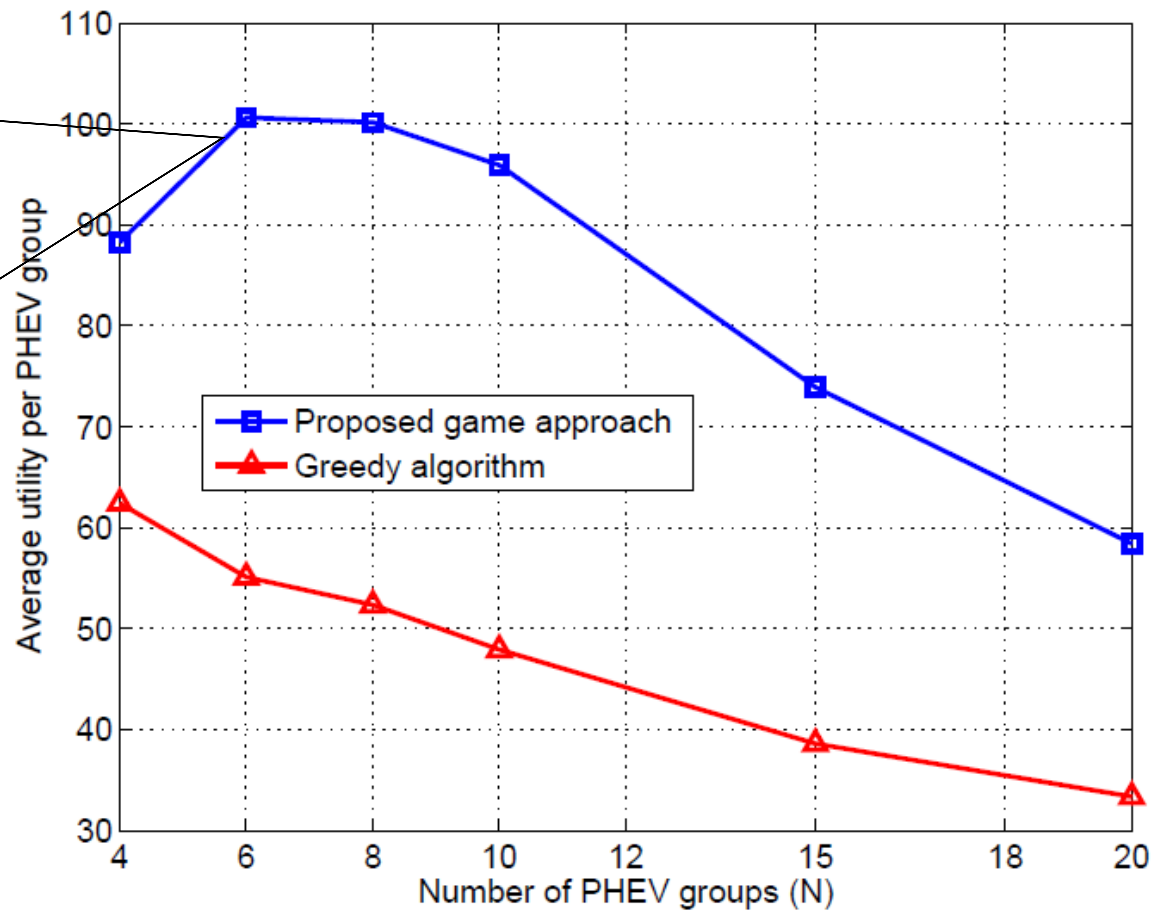
Pricing factor

- How to solve the game and find the Nash equilibrium?
  - Auction introduces a discontinuity => difficult analytically
  - Algorithmic approach (based on best-response)

Typical Simulation Results

- Initially, the utility increases as more players enter the game leading to more energy sold.

- Then, the utility decreases as the presence of more sellers deflates the price.

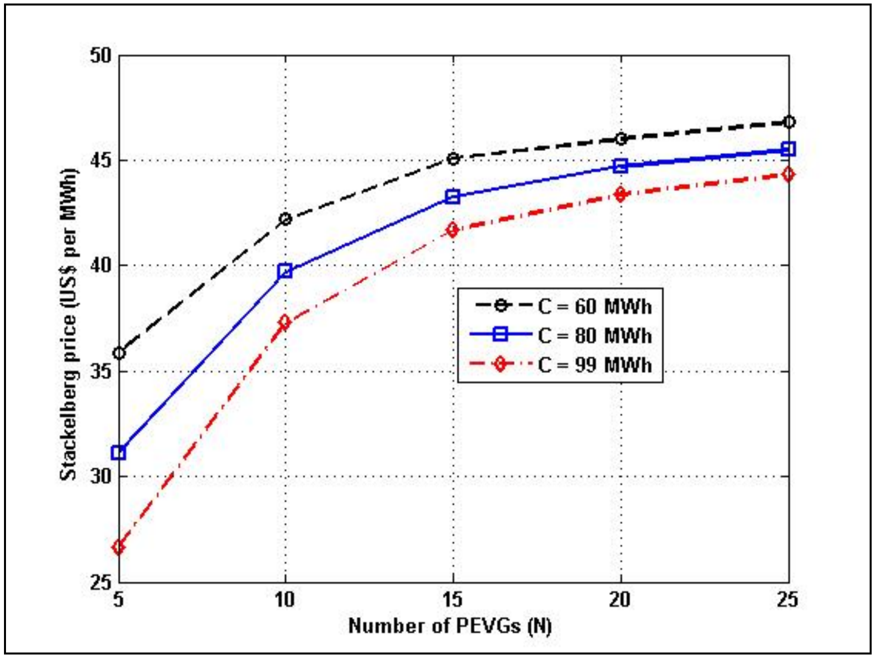


A Stackelberg Model

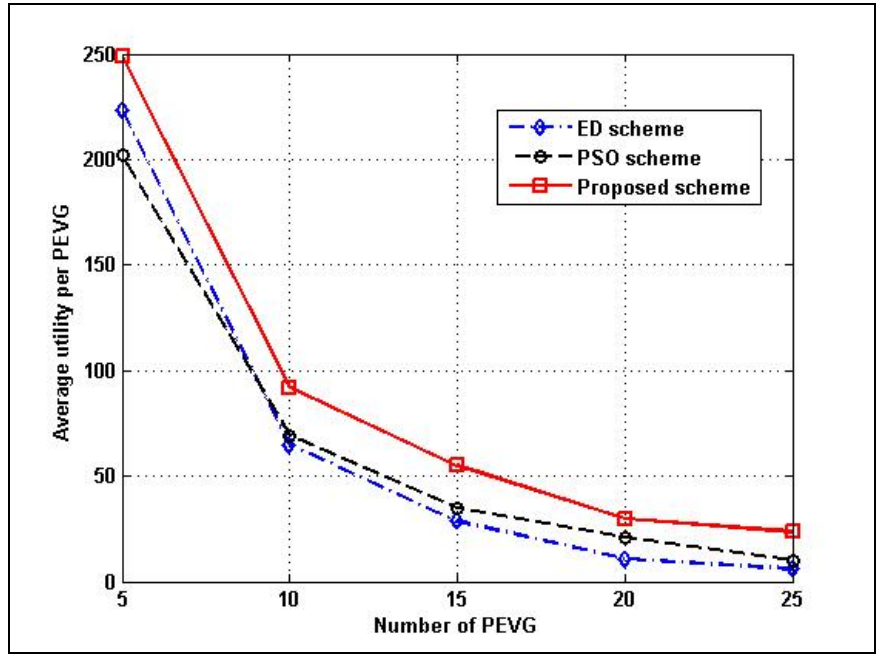
[w/ Tushar, Saad, Smith - T-SG'12]

- Consider now the grid acting as a single entity (and selling to the vehicle groups).
- Then we have a powerful leader (the grid) and less powerful (and competing) followers (the vehicle groups) - a Stackelberg game
- The utilities of the vehicle groups are still linear-quadratic in their strategies (i.e., how much they buy).
- But, the price is set by the leader.
- The leader's utility is bi-linear = price $\times$ total quantity sold.
- Leads to a Stackelberg equilibrium.

Typical Simulation Results



Price vs. # Groups



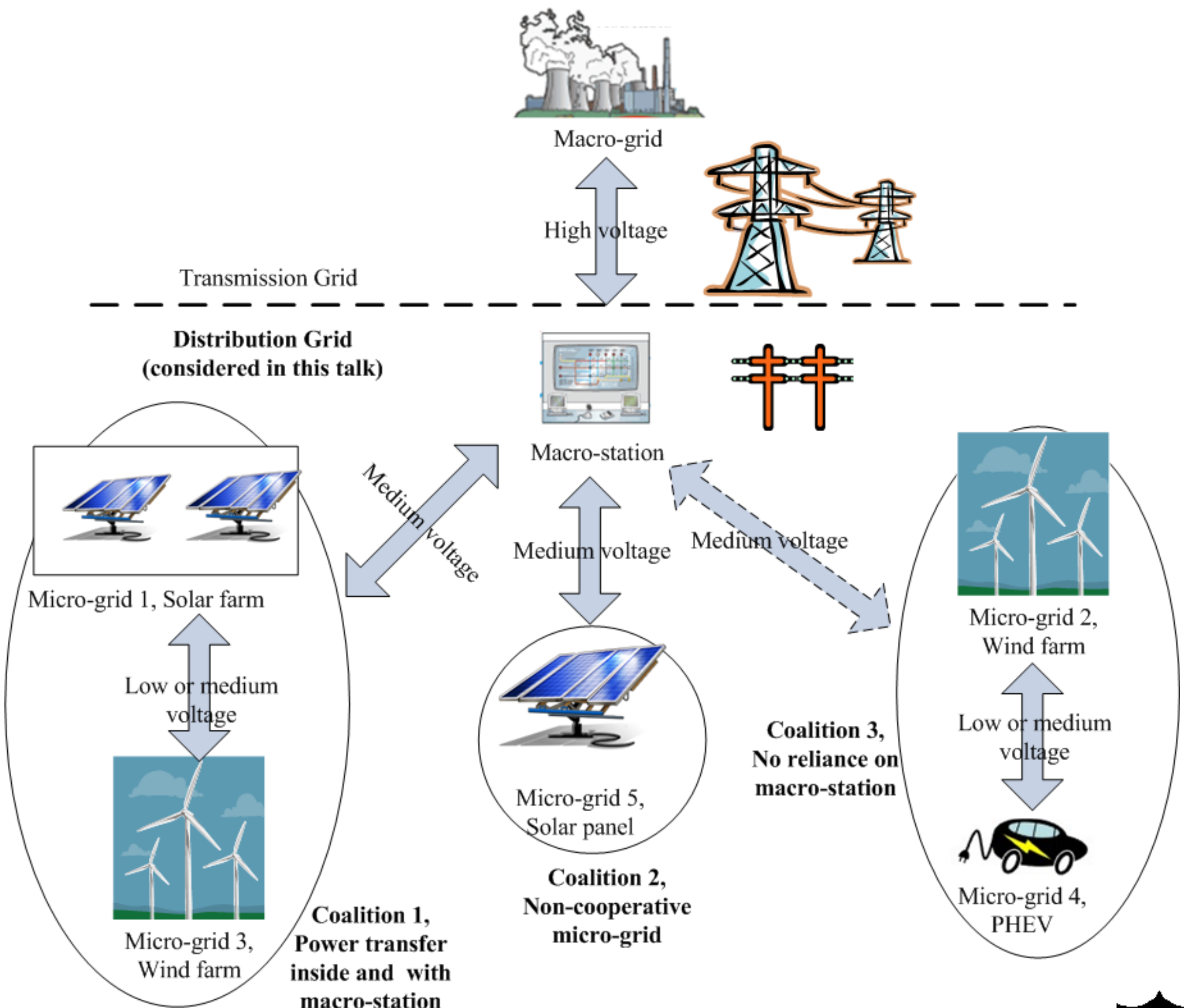
Ave. Utility vs. # Groups*

*PSO = particle swarm optimization
ED = equal distributions

Ex. 2: Micro-grid Interaction

[w/ Saad, Han– ICC'11]

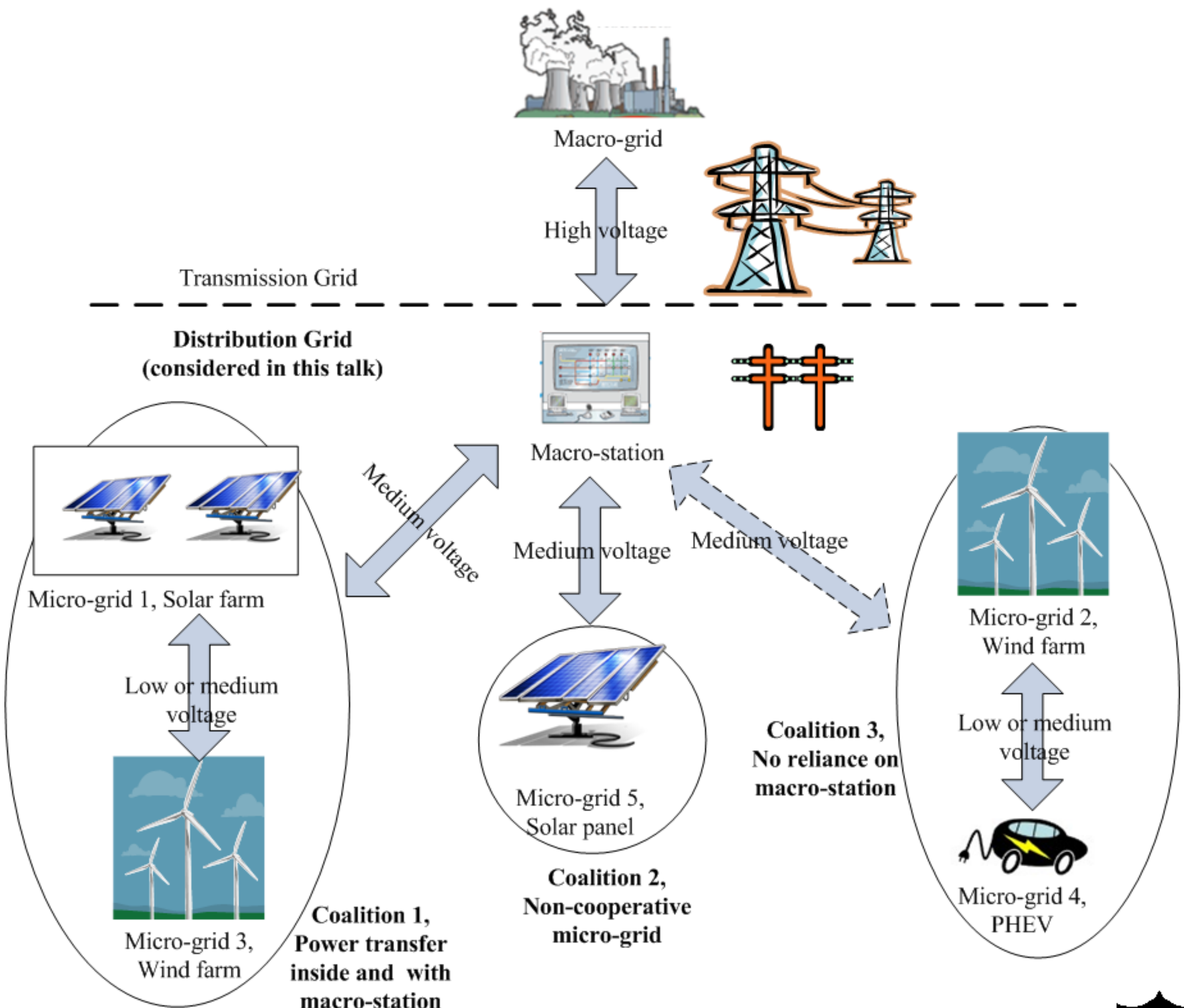
- Energy trading within the distribution network



Ex. 2: Micro-grid Interaction

[w/ Saad, Han– ICC'11]

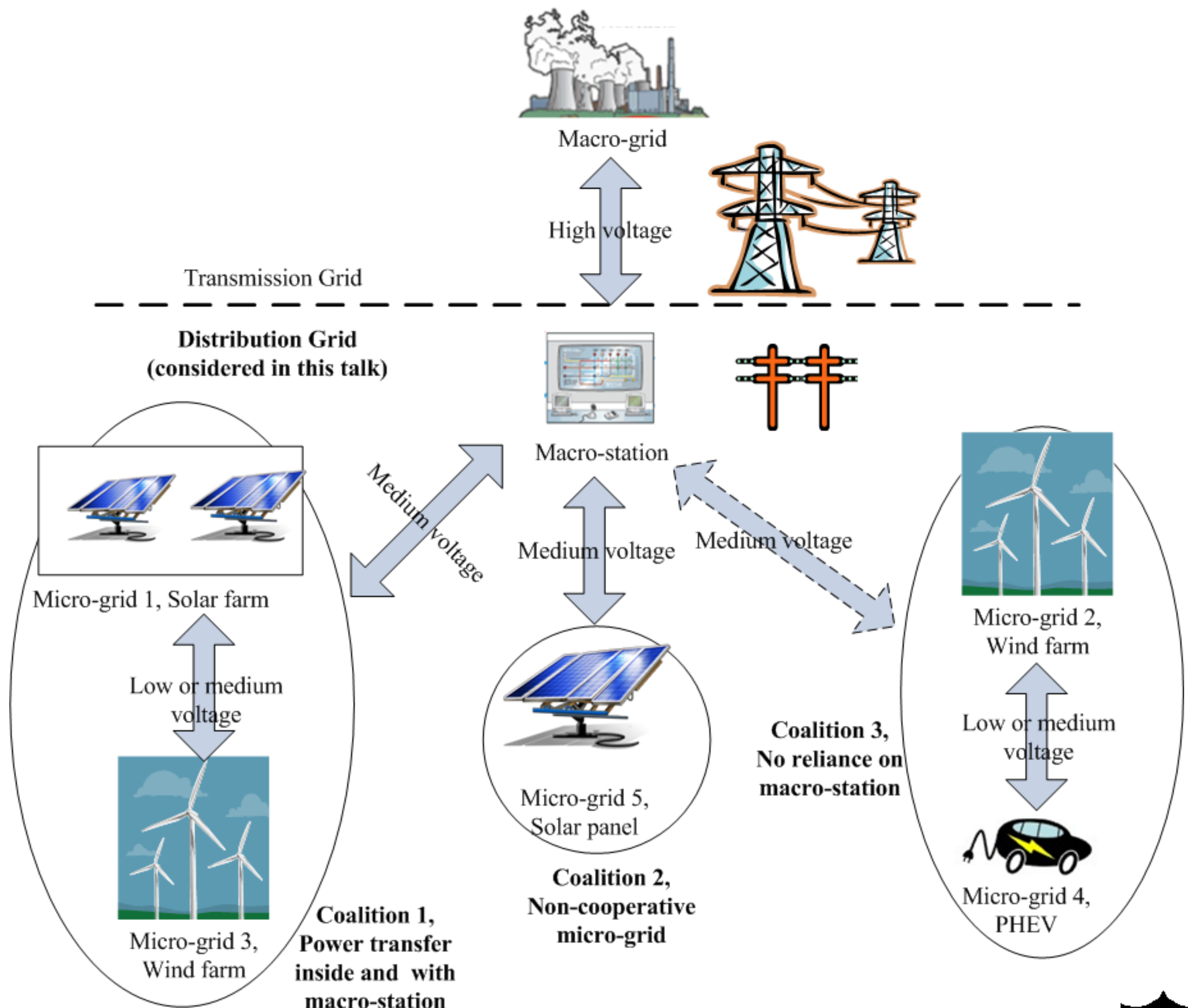
- Energy trading within the distribution network
- Cooperation helps to:
  - Exchange energy: sell surplus and overcome deficiency
  - Reduce power losses over transmission lines



Ex. 2: Micro-grid Interaction

[w/ Saad, Han– ICC'11]

- Energy trading within the distribution network
- Cooperation helps to:
  - Exchange energy: sell surplus and overcome deficiency
  - Reduce power losses over transmission lines
- Coalitional games



Coalition Games

- Coalitional game (N,v)
  - In a set of players N, *a coalition* S is a group of cooperating players
  - Value (utility) of a coalition $v(S)$
  - User payoff $\phi_i(S)$: the portion received by a player i in a coalition S

Coalition Games

- Coalitional game (N,v)
  - In a set of players N, *a coalition* S is a group of cooperating players
  - Value (utility) of a coalition $v(S)$
  - User payoff $\phi_i(S)$: the portion received by a player i in a coalition S

- Coalition formation
  - Coalitions can be compared based on Pareto ordering of user payoffs
  - Merges and splits can be used to iterate on coalitions
  - Convergence to a stable, merge-and-split-proof limit

Game Formulation: Value Function

- For a coalition S, we define the value function as

$$v(S) = \max_{\pi \in \mathfrak{T}_S} u(S, \pi)$$

  - The max is over all orderings of buyers & *u* measures power losses.
  - The utility represents a cost paid per unit of power loss.

Game Formulation: Value Function

- For a coalition S, we define the value function as

$$v(S) = \max_{\pi \in \mathfrak{T}_S} u(S, \pi)$$

  - The max is over all orderings of buyers & *u* measures power losses.
  - The utility represents a cost paid per unit of power loss.

- To divide the utility between the players, adopt a fair division proportional to the non-cooperative utility of each user:

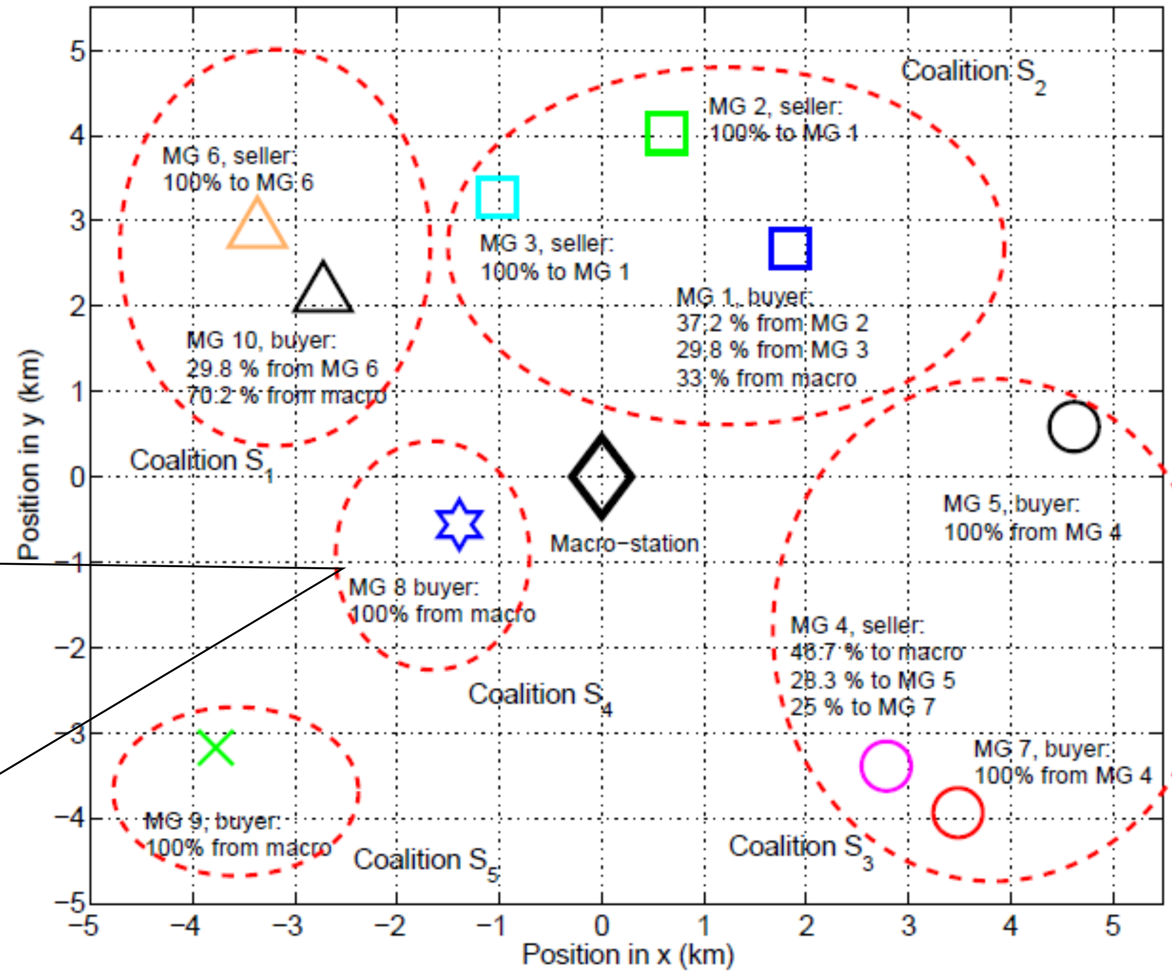
$$\phi_i = \alpha_i \left(v(S) - \sum_{j \in S} v(\{j\}) \right) + v(\{i\}),$$

$$\frac{\alpha_i}{\alpha_j} = \frac{v(\{i\})}{v(\{j\})} \qquad \sum_{i \in S} \alpha_i = 1$$

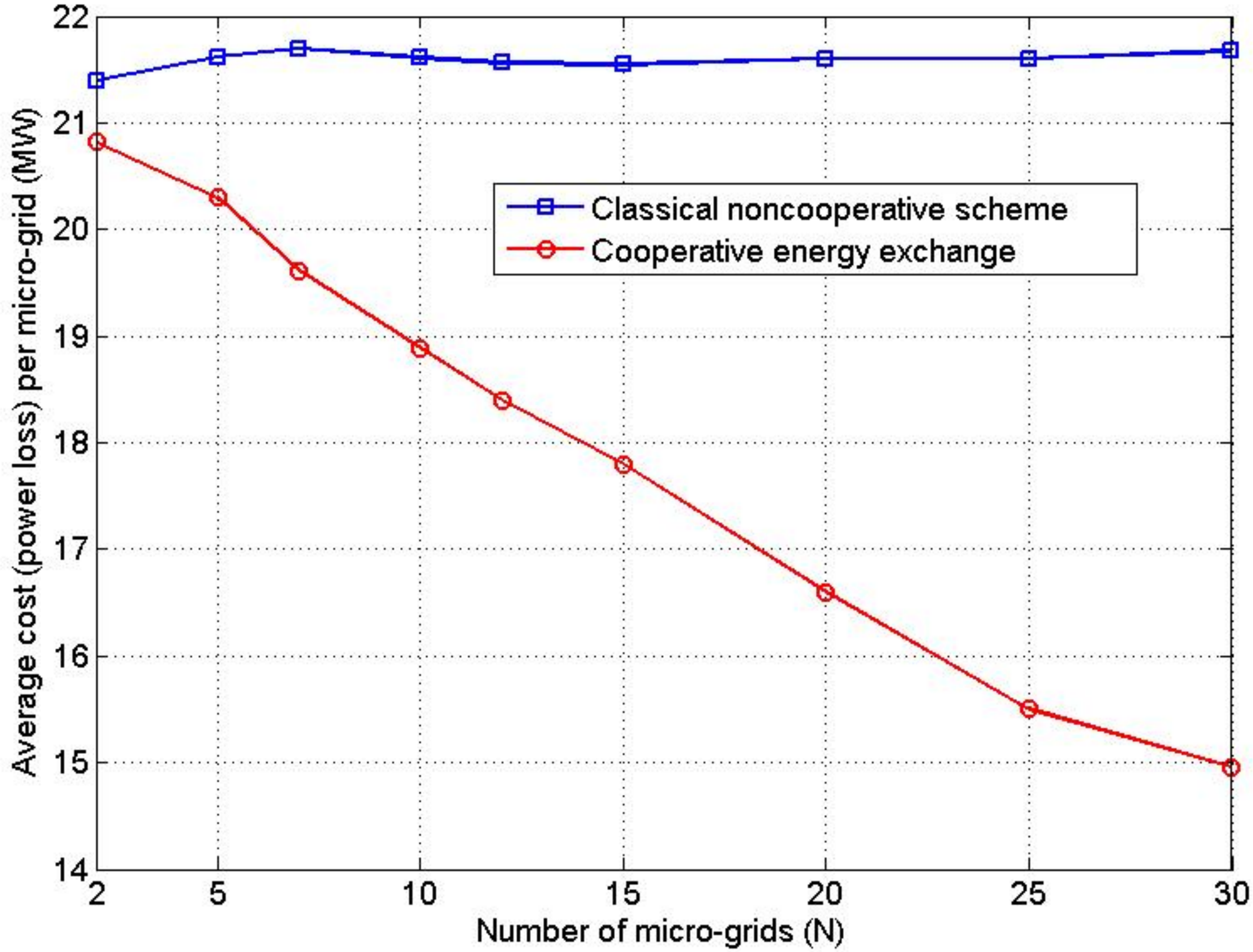
Weight chosen according to micro-grid *i*'s non-cooperative utility

Typical Simulation Results (1)

- Emergence of local markets
- Here, we see a single snapshot; it is of interest for future work to see how this evolves as demand/ supply vary



Typical Simulation Results (2)



Game Theoretic Methods for Modeling Interactions

Summary

- Game theory for smart grid modeling:
  - Demand-side management, energy trading and markets
  - Integration and distributed operation of micro-grids
  - "Game theoretic methods for the smart grid," [w/ Saad, Han, Basar - SPM'12]

Summary

- Game theory for smart grid modeling:
  - Demand-side management, energy trading and markets
  - Integration and distributed operation of micro-grids
  - "Game theoretic methods for the smart grid," [w/ Saad, Han, Basar - SPM'12]
- Other problems of interest
  - Network formation games for PLC backhaul [w/ Saad, Han - Gamenets'11]
  - Social optimality of equilibria in trading markets [w/ Tushar, et al. – ICC'13]

Summary

- Game theory for smart grid modeling:
  - Demand-side management, energy trading and markets
  - Integration and distributed operation of micro-grids
  - "Game theoretic methods for the smart grid," [w/ Saad, Han, Basar - SPM'12]
- Other problems of interest
  - Network formation games for PLC backhaul [w/ Saad, Han - Gamenets'11]
  - Social optimality of equilibria in trading markets [w/ Tushar, et al. – ICC'13]
- Additional issues
  - Optimizing jointly over three layers: economic, cyber, and physical
  - Incorporating dynamics (generation/load/mobility/etc.)

Privacy-Utility Tradeoffs for Data Sources

Joint work with Lalitha Sankar, et al.

Motivation: The Privacy Problem

- There are many electronic information sources of information about us.
  - Google, Facebook, smart metering, etc.

Motivation: The Privacy Problem

- There are many electronic information sources of information about us.
  - Google, Facebook, smart metering, etc.

- The utility of these sources depends on their accessibility.

Motivation: The Privacy Problem

- There are many electronic information sources of information about us.
  - Google, Facebook, smart metering, etc.

- The utility of these sources depends on their accessibility.
- But, they can also leak private information.

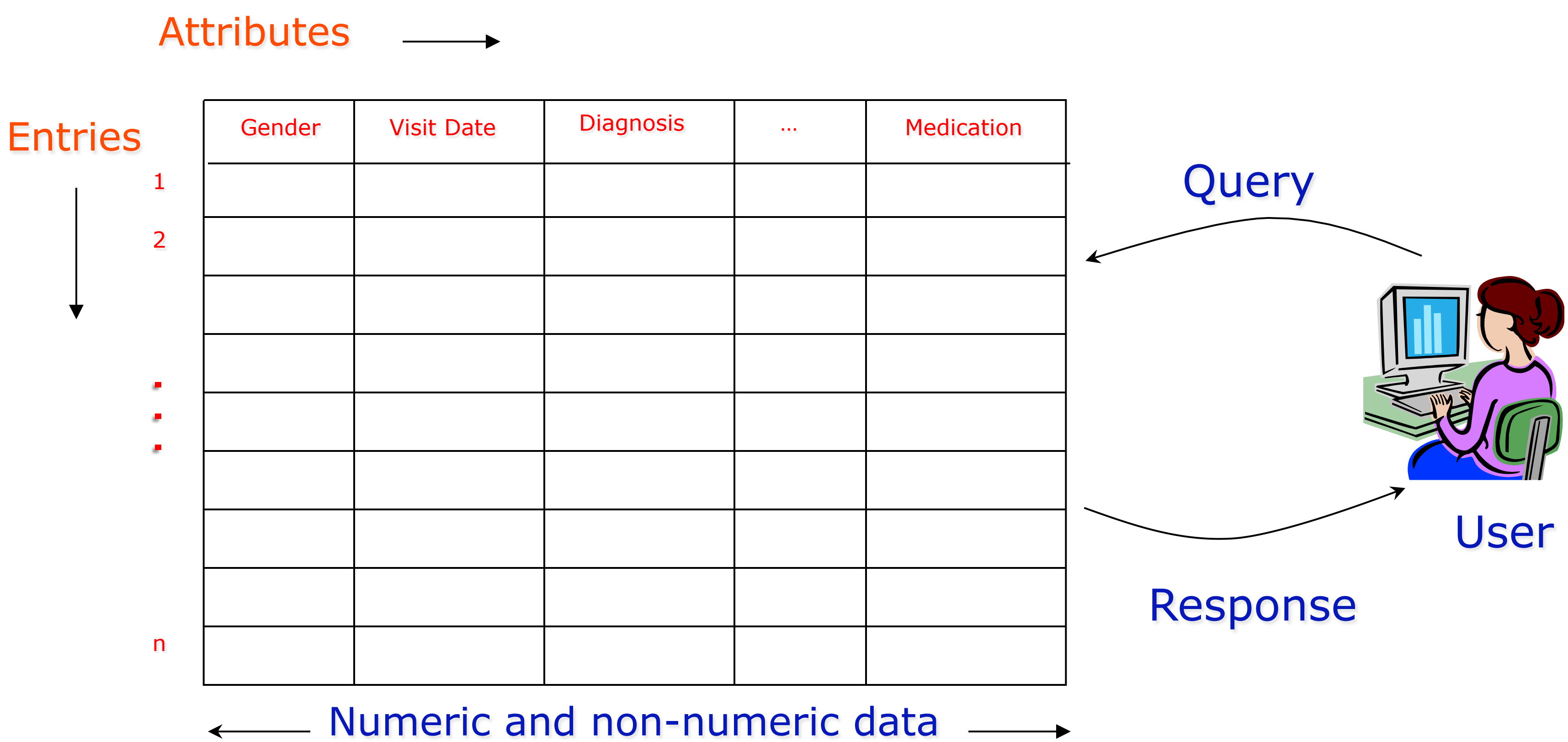
Motivation: The Privacy Problem

- There are many electronic information sources of information about us.
  - Google, Facebook, smart metering, etc.

- The utility of these sources depends on their accessibility.
- But, they can also leak private information.
- How can we characterize this fundamental tradeoff?

Database Model

A database is a table – rows: individual entries (total of *n*);
columns: attributes for each individual (total of *K*)



Database: Source Model

- Database with *n* rows is a sequence of *n* i.i.d. observations of a vector random variable $\mathbf{X} = (X_1\ X_2\ \ldots\ X_K)$ with a joint distribution:

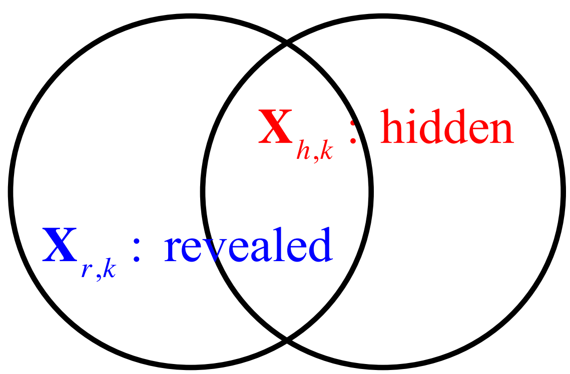
$$\boxed{p_{\mathbf{X}}(\mathbf{x}) = p_{X_1 X_2 \ldots X_K}(x_1, x_2, \ldots, x_K)}$$

Database: Source Model

- Database with *n* rows is a sequence of *n* i.i.d. observations of a vector random variable $\mathbf{X} = (X_1\ X_2\ \ldots\ X_K)$ with a joint distribution:

$$\boxed{p_{\mathbf{X}}(\mathbf{x}) = p_{X_1 X_2 \ldots X_K}(x_1, x_2, \ldots, x_K)}$$

- Attributes divided into public (revealed) and private (hidden) variables, typically not disjoint:



$$k^{th}\ \text{entry} : \mathbf{X}_k = \left(\mathbf{X}_{r,k}, \mathbf{X}_{h,k}\right)$$

Privacy-Utility Tradeoff

[w/ Sankar, Rajagapolan - T-IFS’13]

- Contrast between privacy and secrecy:

Privacy-Utility Tradeoff

[w/ Sankar, Rajagapolan - T-IFS'13]

- Contrast between privacy and secrecy:

  – In the (communications) secrecy problem, there is a single source with legitimate and eavesdropping receivers.

Privacy-Utility Tradeoff

[w/ Sankar, Rajagapolan - T-IFS'13]

- Contrast between privacy and secrecy:
  - In the (communications) secrecy problem, there is a single source with legitimate and eavesdropping receivers.
  - In the privacy problem, we have a single "receiver" (the query initiator) with the source being divided into private and public variables.

Privacy-Utility Tradeoff

[w/ Sankar, Rajagapolan - T-IFS'13]

- Contrast between privacy and secrecy:
  - In the (communications) secrecy problem, there is a single source with legitimate and eavesdropping receivers.
  - In the privacy problem, we have a single "receiver" (the query initiator) with the source being divided into private and public variables.
- How can we characterize the tradeoff between utility and privacy in such a setting?

Privacy-Utility Tradeoff

[w/ Sankar, Rajagapolan - T-IFS'13]

- Contrast between privacy and secrecy:
  - In the (communications) secrecy problem, there is a single source with legitimate and eavesdropping receivers.
  - In the privacy problem, we have a single "receiver" (the query initiator) with the source being divided into private and public variables.
- How can we characterize the tradeoff between utility and privacy in such a setting?
  - Measure utility by distortion of the public variables as revealed to a user of the database;

Privacy-Utility Tradeoff

[w/ Sankar, Rajagapolan - T-IFS'13]

- Contrast between privacy and secrecy:
  - In the (communications) secrecy problem, there is a single source with legitimate and eavesdropping receivers.
  - In the privacy problem, we have a single "receiver" (the query initiator) with the source being divided into private and public variables.
- How can we characterize the tradeoff between utility and privacy in such a setting?
  - Measure utility by distortion of the public variables as revealed to a user of the database; and
  - Measure privacy by equivocation on the private variables in information revealed to a user.

Privacy-Utility Tradeoff

[w/ Sankar, Rajagapolan - T-IFS'13]

- Contrast between privacy and secrecy:
  - In the (communications) secrecy problem, there is a single source with legitimate and eavesdropping receivers.
  - In the privacy problem, we have a single "receiver" (the query initiator) with the source being divided into private and public variables.
- How can we characterize the tradeoff between utility and privacy in such a setting?
  - Measure utility by distortion of the public variables as revealed to a user of the database; and
  - Measure privacy by equivocation on the private variables in information revealed to a user.
- Then the distortion-equivocation region describes the tradeoff.

Distortion-Equivocation Model

[w/ Sankar, Rajagapolan - T-IFS'13]

- Encoder maps the original database to a "sanitized" database (SDB):

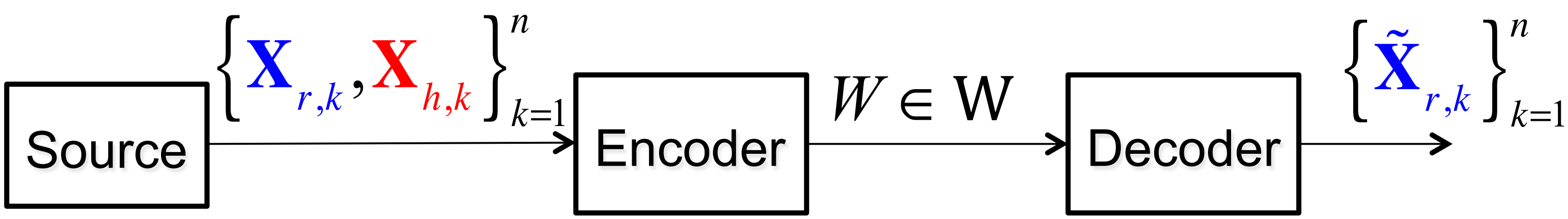
$$\text{Encoder}: \mathbf{X}^n \rightarrow \mathcal{W} = \left\{SDB_1, SDB_2, \ldots, SDB_M\right\}$$

Distortion-Equivocation Model

[w/ Sankar, Rajagapolan - T-IFS'13]

- Encoder maps the original database to a "sanitized" database (SDB):

$$\text{Encoder}:\mathbf{X}^n \to \mathcal{W} = \left\{SDB_1, SDB_2, \ldots, SDB_M\right\}$$



Distortion-Equivocation Model

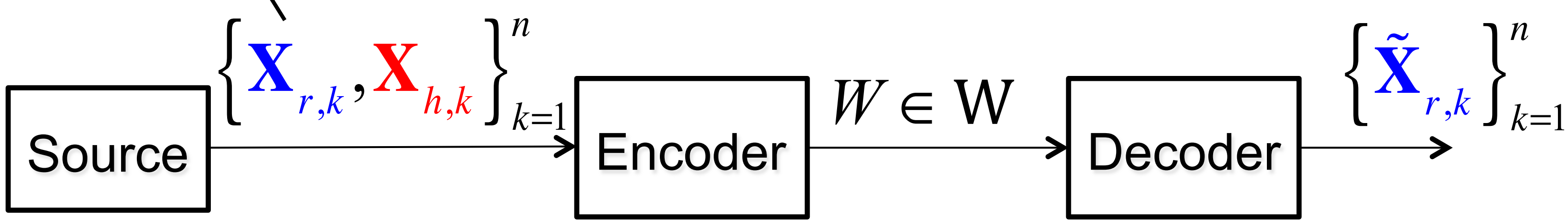
[w/ Sankar, Rajagapolan - T-IFS'13]

- Encoder maps the original database to a "sanitized" database (SDB):

$$\text{Encoder}: \mathbf{X}^n \rightarrow \mathcal{W} = \left\{ SDB_1, SDB_2, \ldots, SDB_M \right\}$$

Distortion

$$\Delta_d \equiv \mathbb{E}\left[\frac{1}{n} \sum_{i=1}^{n} \rho\left(\mathbf{X}_{r,i}, \tilde{\mathbf{X}}_{r,i} \right) \right] \leq D + \varepsilon$$



Privacy-Utility Tradeoffs for Data Sources

Distortion-Equivocation Model

[w/ Sankar, Rajagapolan - T-IFS'13]

- Encoder maps the original database to a "sanitized" database (SDB):

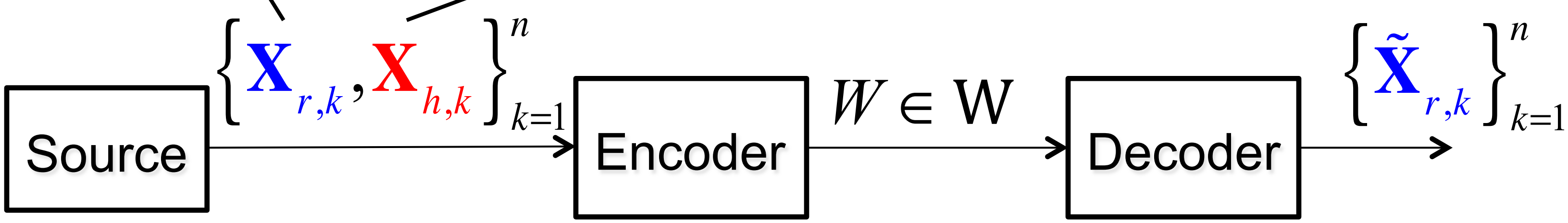
$$\text{Encoder} : \mathbf{X}^n \rightarrow \mathcal{W} = \left\{ SDB_1, SDB_2, \ldots, SDB_M \right\}$$

Distortion

$$\Delta_d \equiv \mathbb{E}\left[\frac{1}{n} \sum_{i=1}^{n} \rho\left(\mathbf{X}_{r,i}, \tilde{\mathbf{X}}_{r,i} \right) \right] \leq D + \varepsilon$$

Equivocation

$$\Delta_p \equiv \frac{1}{n} H\left(\mathbf{X}_h^n \,|\, W \right) > E - \varepsilon$$



Distortion-Equivocation Model

[w/ Sankar, Rajagapolan - T-IFS'13]

- Encoder maps the original database to a "sanitized" database (SDB):

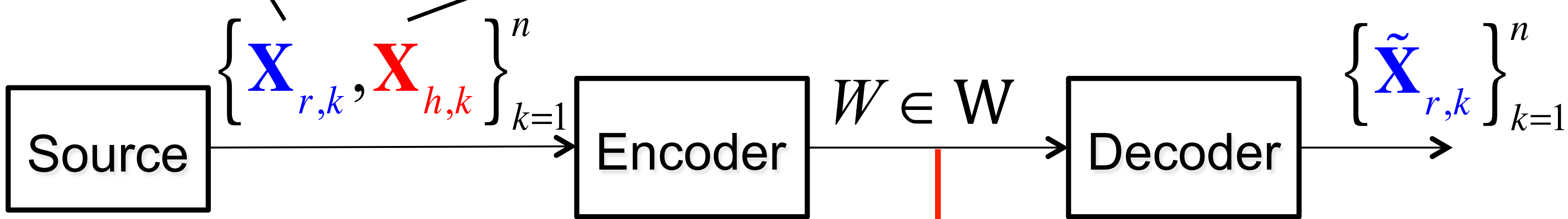
$$\text{Encoder}: \mathbf{X}^n \to \mathcal{W} = \{SDB_1, SDB_2, \ldots, SDB_M\}$$

Distortion

$$\Delta_d \equiv \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} \rho\left(\mathbf{X}_{r,i}, \tilde{\mathbf{X}}_{r,i}\right)\right] \le D + \varepsilon$$

Equivocation

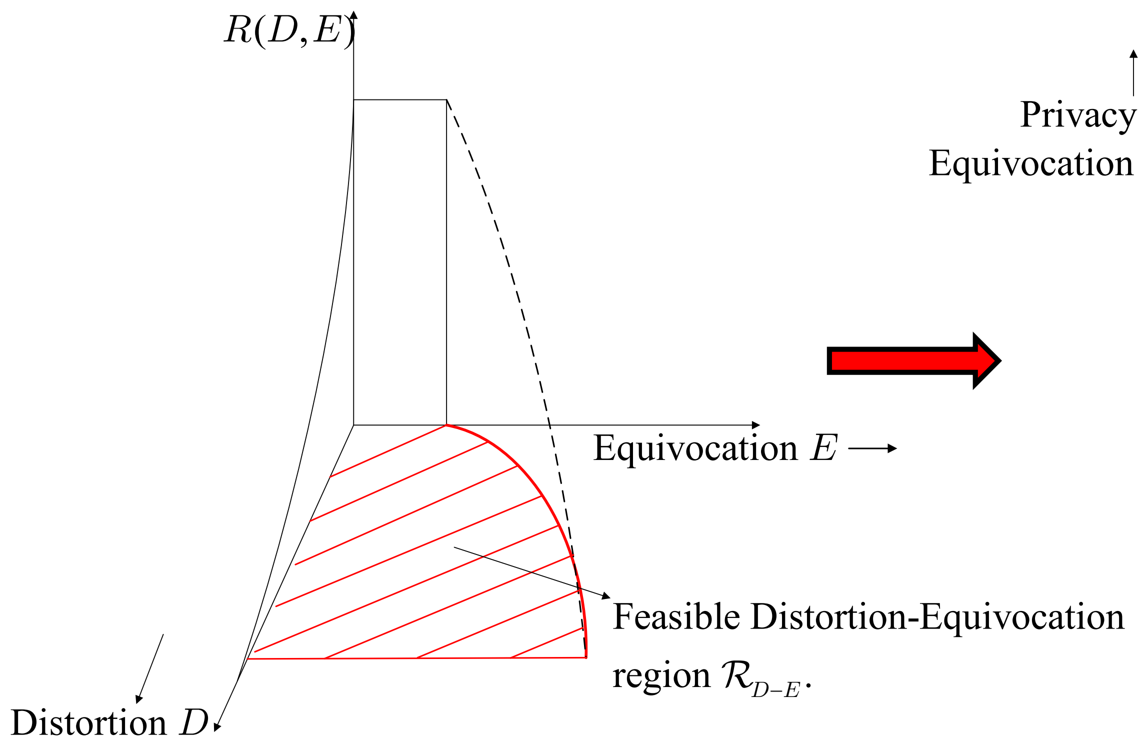
$$\Delta_p \equiv \frac{1}{n} H\left(\mathbf{X}_h^n \,|\, W\right) > E - \varepsilon$$



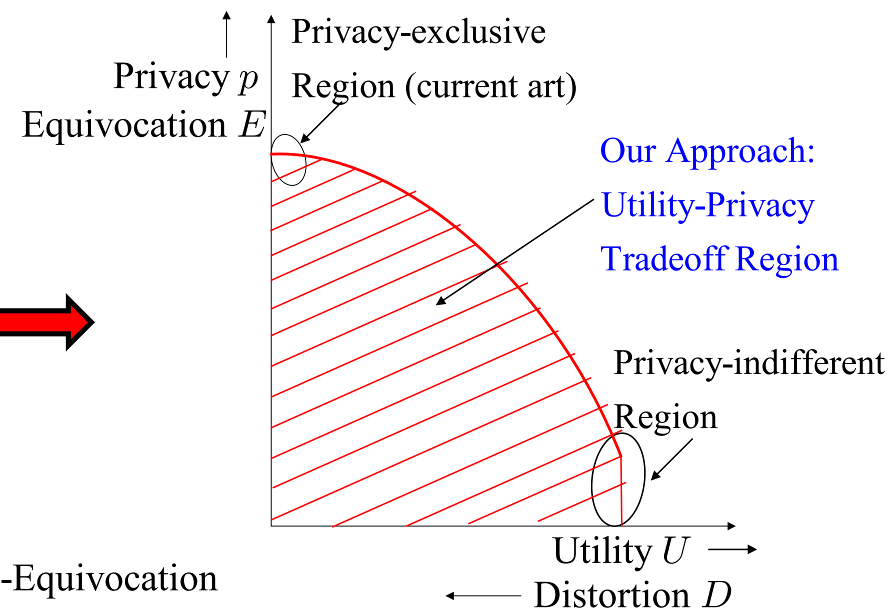
Add a rate constraint → $M \le 2^{n(R+\varepsilon)}$

Privacy-Utility Tradeoffs for Data Sources

Utility-Privacy/RDE Regions



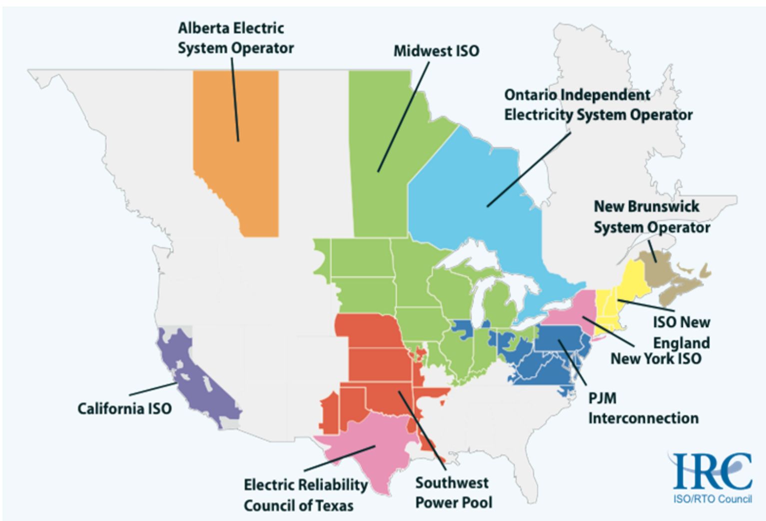
(a): Rate-Distortion-Equivocation Region

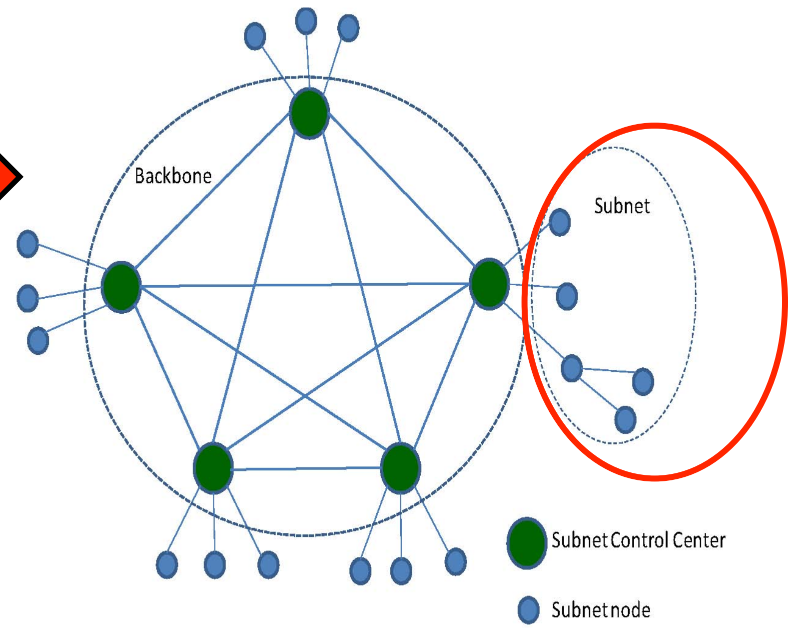


(b): Utility-Privacy Tradeoff Region

Competitive Privacy

- N.A. Grid: interconnected regional transmission organizations (RTOs) which
  - need to share measurements on state estimation for reliability (utility)

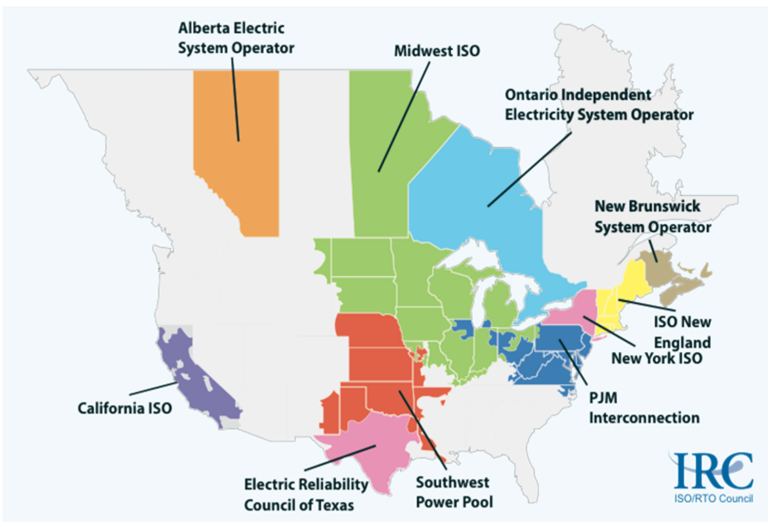


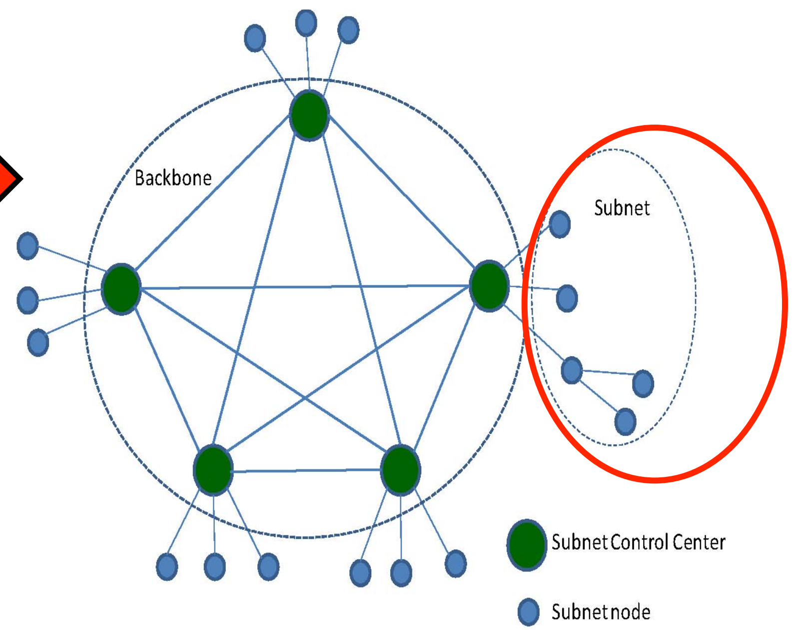


Privacy-Utility Tradeoffs for Data Sources

Competitive Privacy

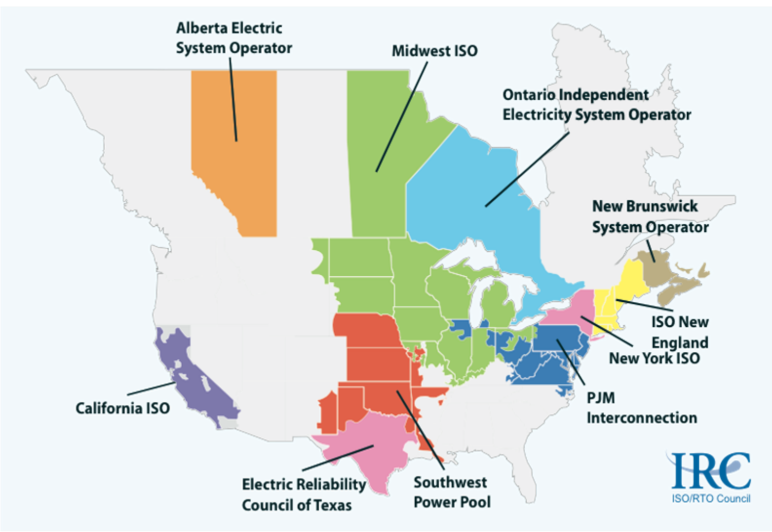
- N.A. Grid: interconnected regional transmission organizations (RTOs) which
  - need to share measurements on state estimation for reliability (utility)
  - wish to withhold information for economic competitive reasons (privacy)

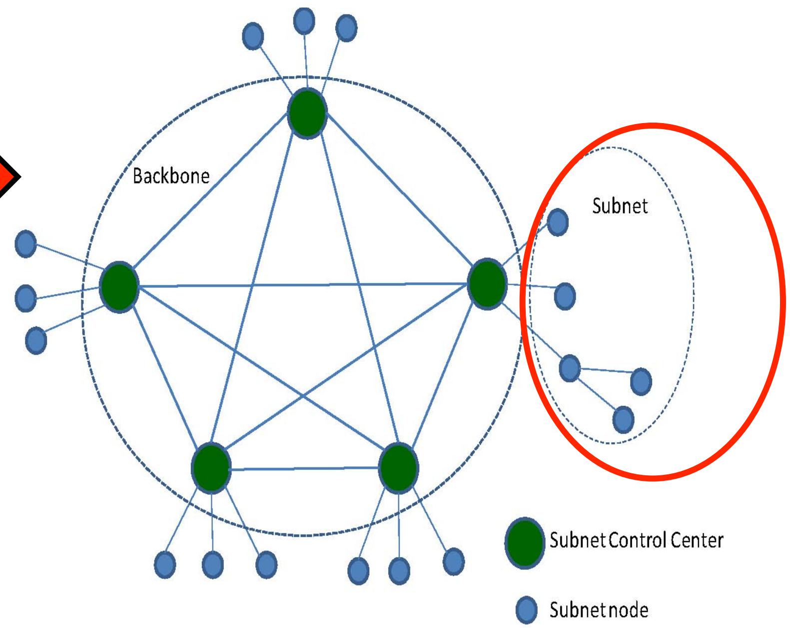




Competitive Privacy

- N.A. Grid: interconnected regional transmission organizations (RTOs) which
  - need to share measurements on state estimation for reliability (utility)
  - wish to withhold information for economic competitive reasons (privacy)





- Leads to a problem of *competitive privacy*

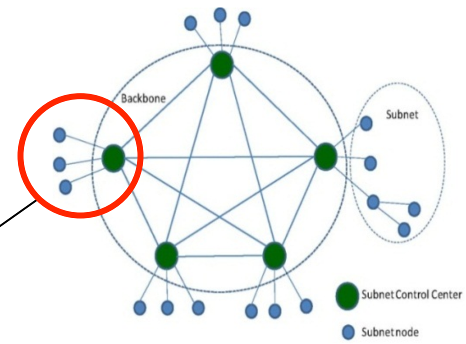
Competitive Privacy

[w /Sankar, Kar - Asilomar'12]

- Noisy measurements at RTO k:

$$Y_k = \sum_{m=1}^{M} H_{k,m} X_m + Z_k,\ k = 1,2,\ldots,M$$

m^{th} system state



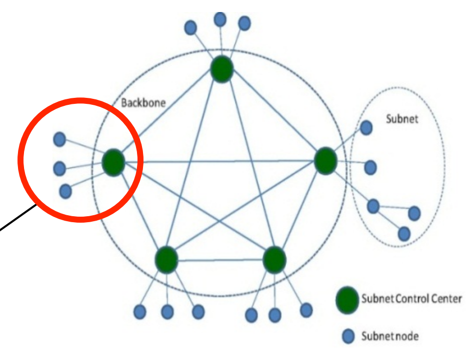
Competitive Privacy

[w /Sankar, Kar - Asilomar'12]

- Noisy measurements at RTO k:

$$Y_k = \sum_{m=1}^{M} H_{k,m} X_m + Z_k, \; k = 1,2,\ldots,M$$

m^{th} system state



- Utility for RTO k: mean-square error for its own state X_k

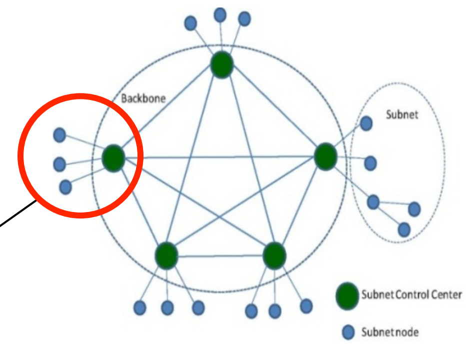
Competitive Privacy

[w /Sankar, Kar - Asilomar'12]

- Noisy measurements at RTO k:

$$Y_k = \sum_{m=1}^{M} H_{k,m} X_m + Z_k,\ k = 1,2,\ldots,M$$

m^{th} system state



- Utility for RTO k: mean-square error for its own state X_k
- Privacy for RTO k: leakage of information about X_k to other RTOs

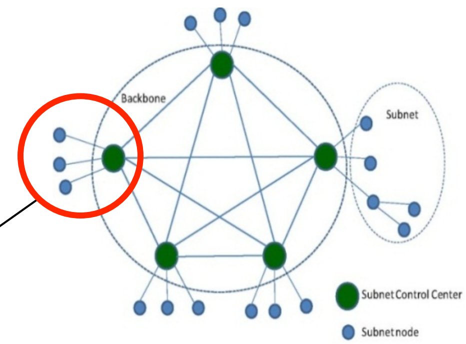
Competitive Privacy

[w /Sankar, Kar - Asilomar'12]

- Noisy measurements at RTO k:

$$Y_k = \sum_{m=1}^{M} H_{k,m} X_m + Z_k \, , \; k = 1,2,\ldots,M$$

m^{th} system state



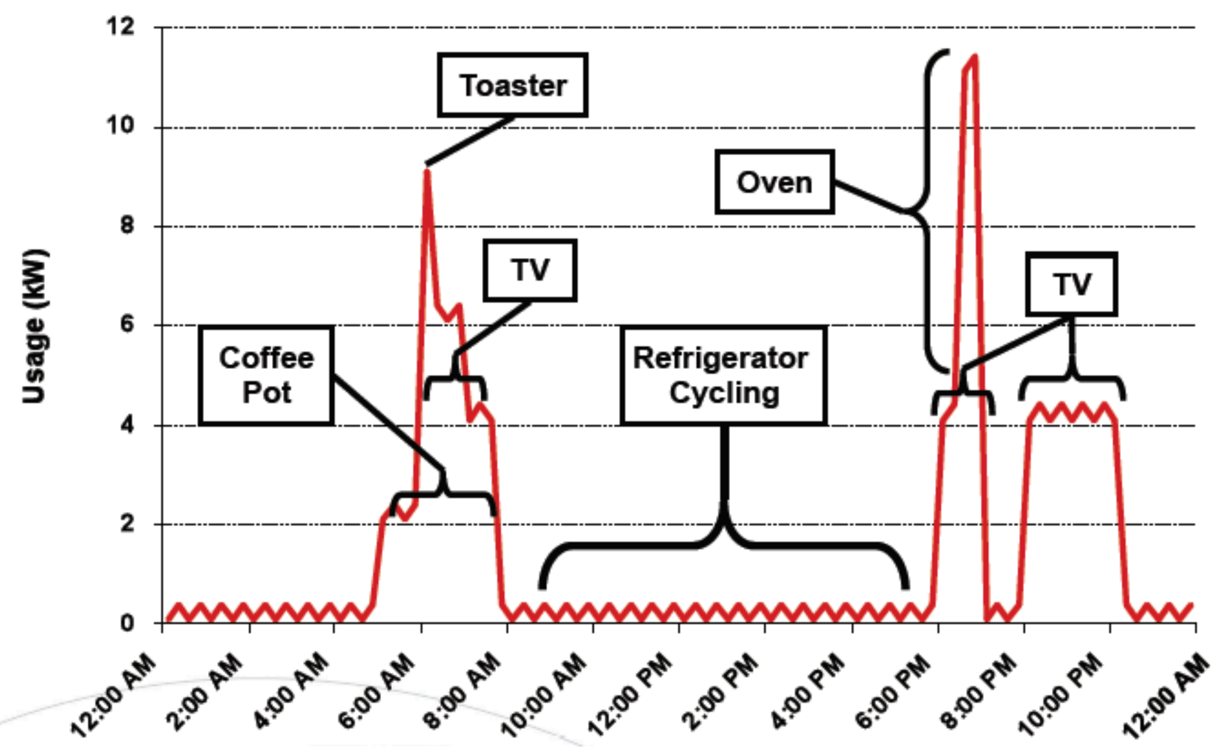
- Utility for RTO k: mean-square error for its own state X_k

- Privacy for RTO k: leakage of information about X_k to other RTOs

Wyner-Ziv coding maximizes privacy for a desired utility at each RTO.

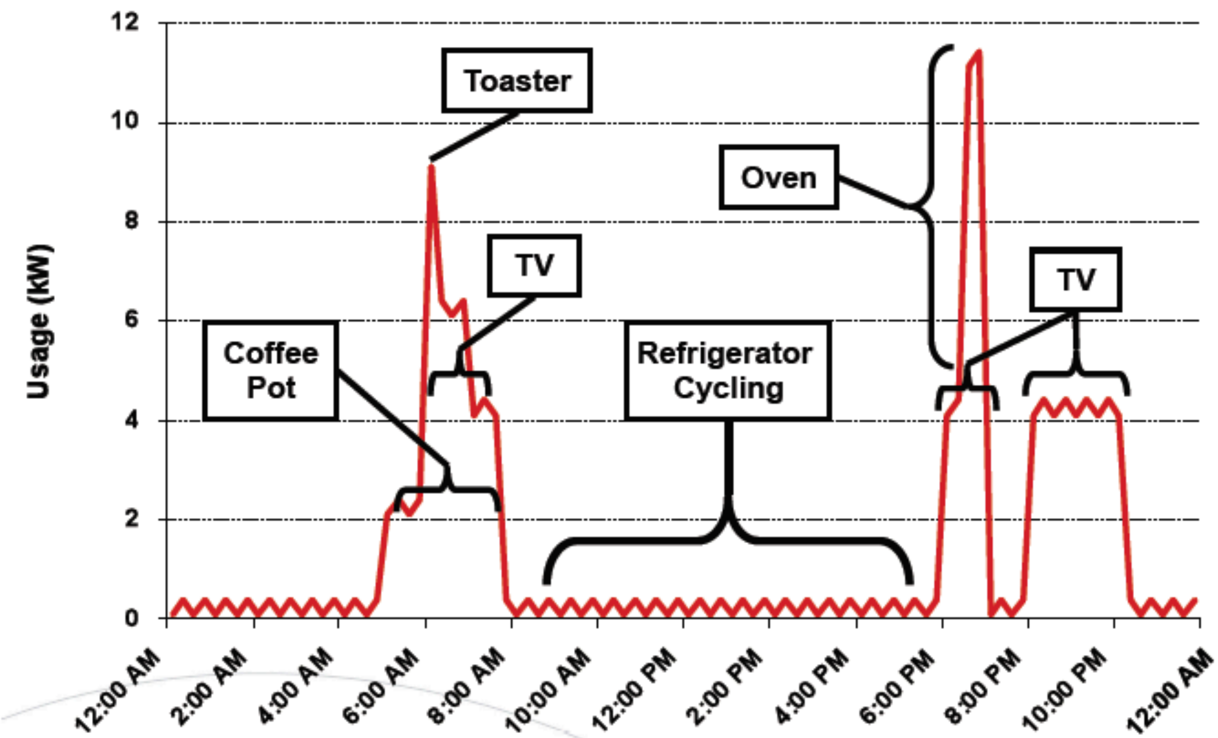
Smart Meter Privacy

- Smart meter data is useful for price-aware usage, load balancing



Smart Meter Privacy

- Smart meter data is useful for price-aware usage, load balancing
- But, it leaks information about in-home activity



Smart Meter Privacy

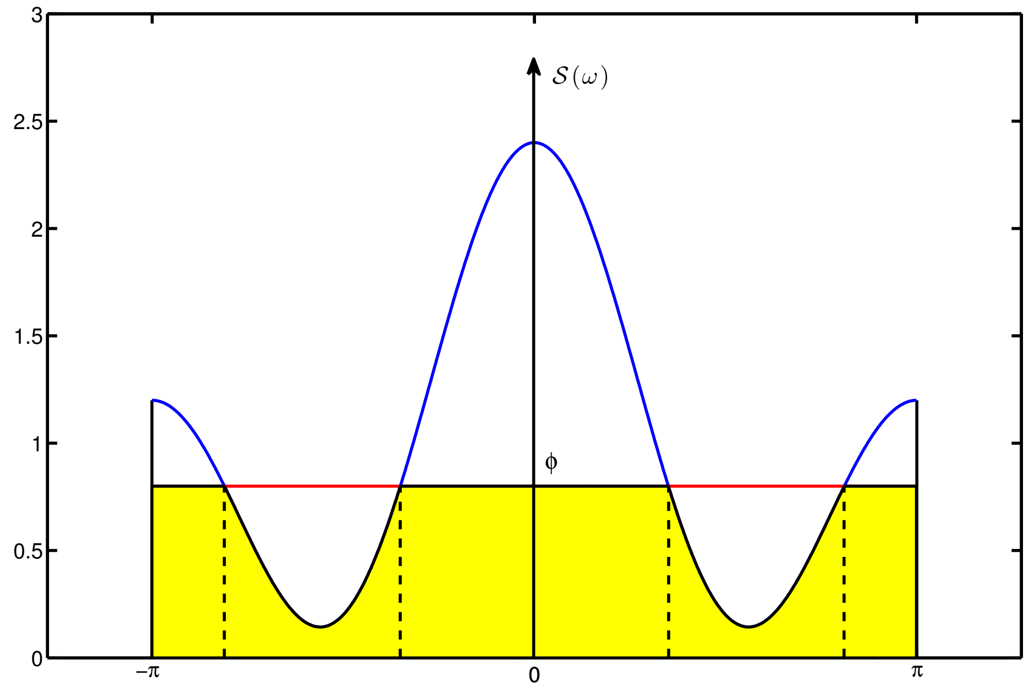
[w /Sankar, Rajagapolan, Mohajer - T-SG'13]

P-U tradeoff leads to a spectral 'reverse water-filling' solution

Smart Meter Privacy

[w /Sankar, Rajagapolan, Mohajer - T-SG'13]

P-U tradeoff leads to a spectral 'reverse water-filling' solution

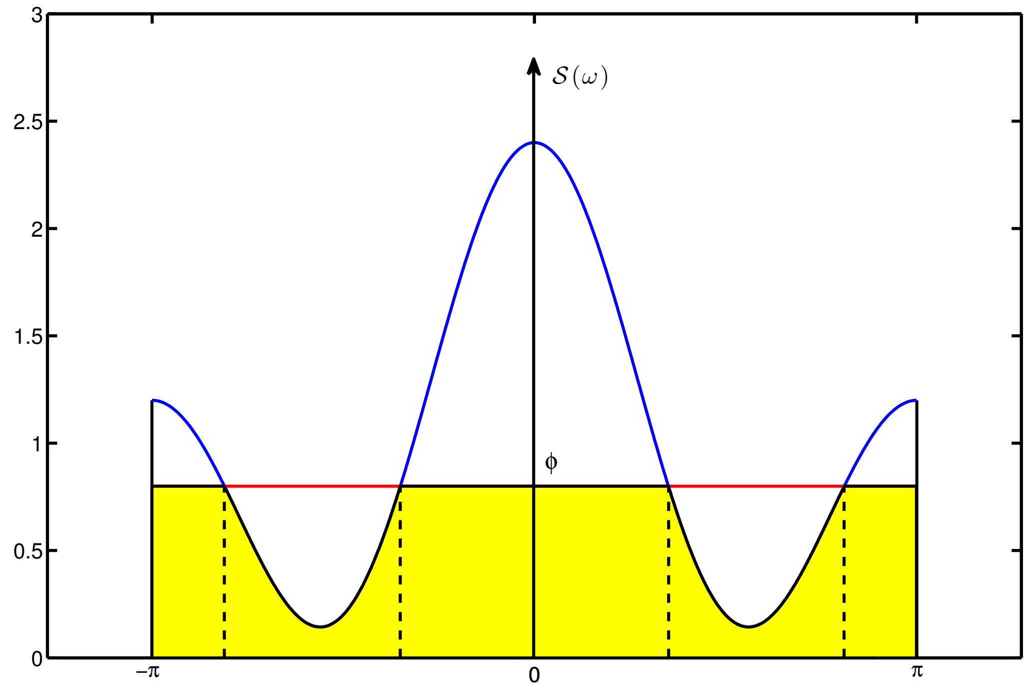


Privacy-Utility Tradeoffs for Data Sources

Smart Meter Privacy

[w /Sankar, Rajagapolan, Mohajer - T-SG'13]

P-U tradeoff leads to a spectral 'reverse water-filling' solution



Can also use energy storage to aid privacy [w/ Tan, Gunduz, JSAC:SG Series'13]

Summary

- An information source is divided into private and public variables
  - Leads to an equivocation-distortion characterization
  - Adding rate: a rate-distortion problem with an equivocation constraint

Summary

- An information source is divided into private and public variables
  - Leads to an equivocation-distortion characterization
  - Adding rate: a rate-distortion problem with an equivocation constraint
- Applications in smart grid include: competitive privacy & smart metering

Summary

- An information source is divided into private and public variables
  - Leads to an equivocation-distortion characterization
  - Adding rate: a rate-distortion problem with an equivocation constraint
- Applications in smart grid include: competitive privacy & smart metering
- Can also consider
  - multiple queries (successive disclosure)
  - multiple sources (side information)

Distributed Algorithms for State Estimation

Joint work with Le Xie, et al.

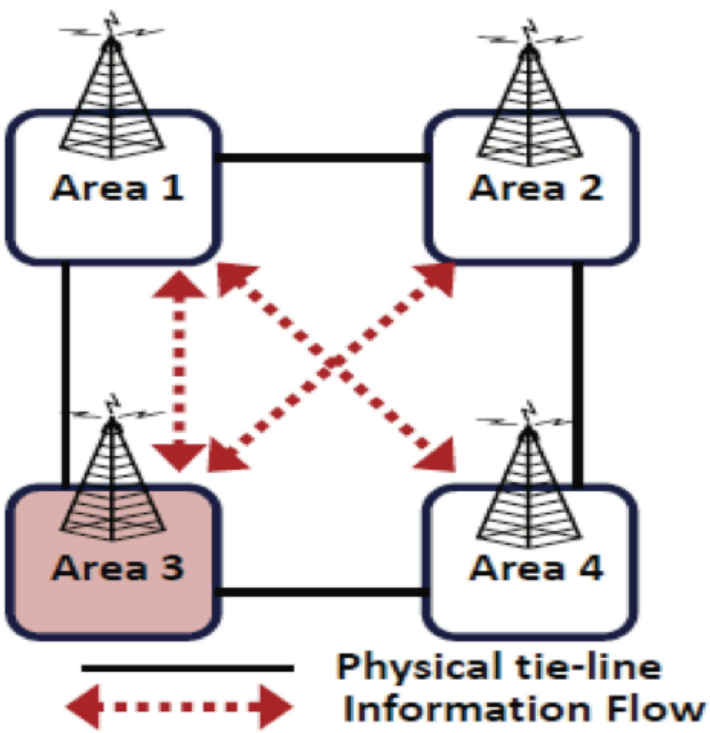
Motivation

- Computational & communications challenge:
  - fast sensing (e.g., Phasor Measurement Units) produces big data, and communications bottlenecks
- Restructuring/deregulation means more RTOs, or control areas (CAs)
- Situational awareness needed for large interconnected power systems:
  - wide area monitoring, control and protection (WAMCP)
- Of interest: a distributed estimation framework to obtain the system-wide states through information exchange among CAs.

Proposed Solution

Wide area state estimation via distributed iterative information processing:

Conceptual Model



Key Properties

- No central coordinator
- Only local information (measurement Jacobian matrix, measurement vector) required
- All local control areas not necessarily observable
- Flexible in communication topology
- Equivalent performance to centralized approach

Distributed Measurement Model

- **System State**
  - $\theta \in \mathbb{R}^M$: The network system state (vector) consisting of voltage phase angles of buses in all CAs.

- **CA Local Observation Model**
  - $\mathbf{z}_n \in \mathbb{R}^{M_n}$: The local observation at CA n

$$\mathbf{z}_n = H_n\theta + \mathbf{e}_n,$$

where the Jacobian $H_n \in \mathbb{R}^{M_n}$ sub-block represents the local physical interconnections.

Proposed Distributed Iterative Solution

[w / Xie, Choi, Kar - T-SG'12]

Each CA n has only local knowledge of the network structure and measurements and updates a local estimate $\mathbf{x}_n$ as follows:

$$\mathbf{x}_n(t+1) = \mathbf{x}_n(t) - \beta_t \sum_{l \in \Omega_n} (\mathbf{x}_n(t) - \mathbf{x}_l(t)) + \alpha_t \overline{H}_n^T \left(\overline{\mathbf{z}}_n - \overline{H}_n \mathbf{x}_n(t) \right),$$

where

- Ω_n: communication neighborhood of CA n
- $\overline{H}_n = R_n^{-1/2} H_n$
- $\overline{\mathbf{z}}_n = R_n^{-1/2} \mathbf{z}_n$

Convergence to Global Estimates

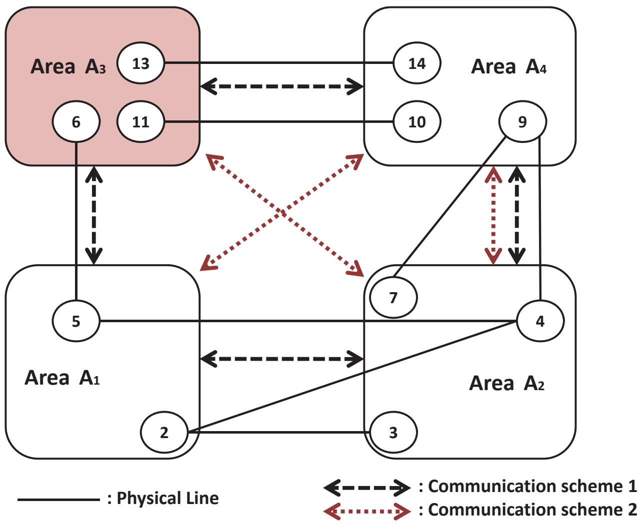
[w / Xie, Choi, Kar - T-SG'12]

Global observability of the grid (i.e., $\sum_{n=1}^{N} H_n^T H_n$ is full rank)

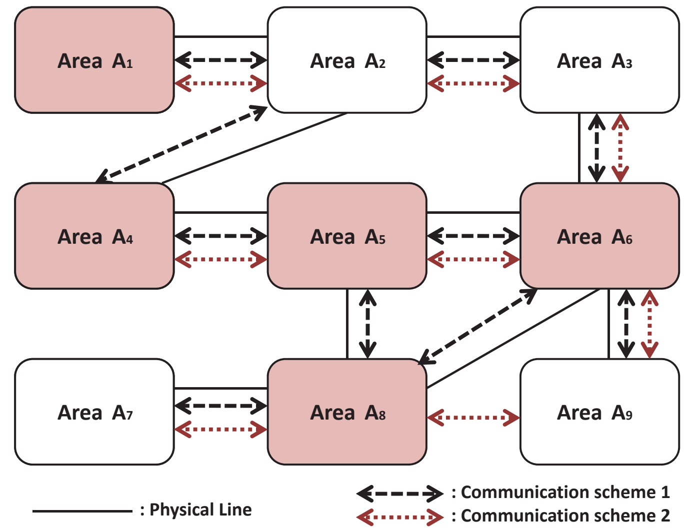
+ connectivity of the communication network (i.e. the second smallest eigenvalue of the graph Laplacian is positive) …

assures a.s. convergence of local estimates to the global estimate (least squares with all measurements) with appropriately programmed α's and β's.

Test Bus Systems



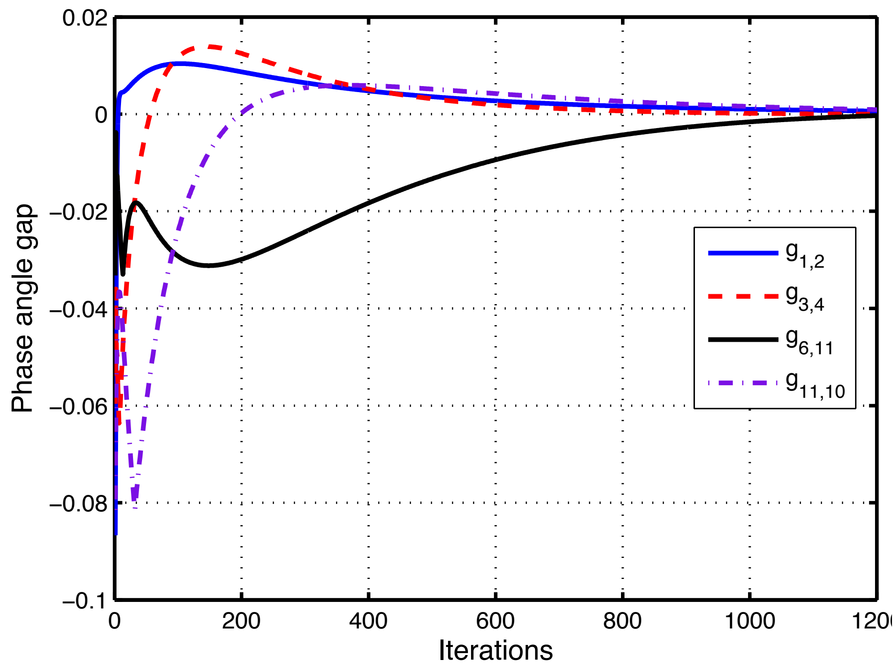
(a) The IEEE 14-bus system



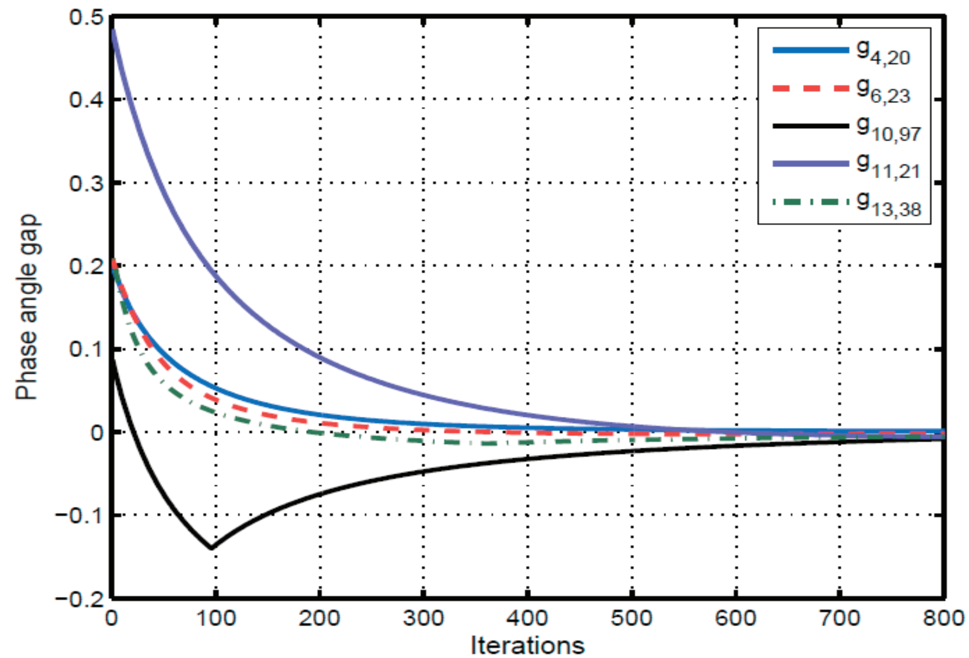
(b) The IEEE 118-bus system

- Overall systems are globally observable
- CAs are globally unobservable
- Shaded CAs are locally unobservable

Convergence of Phase Estimates

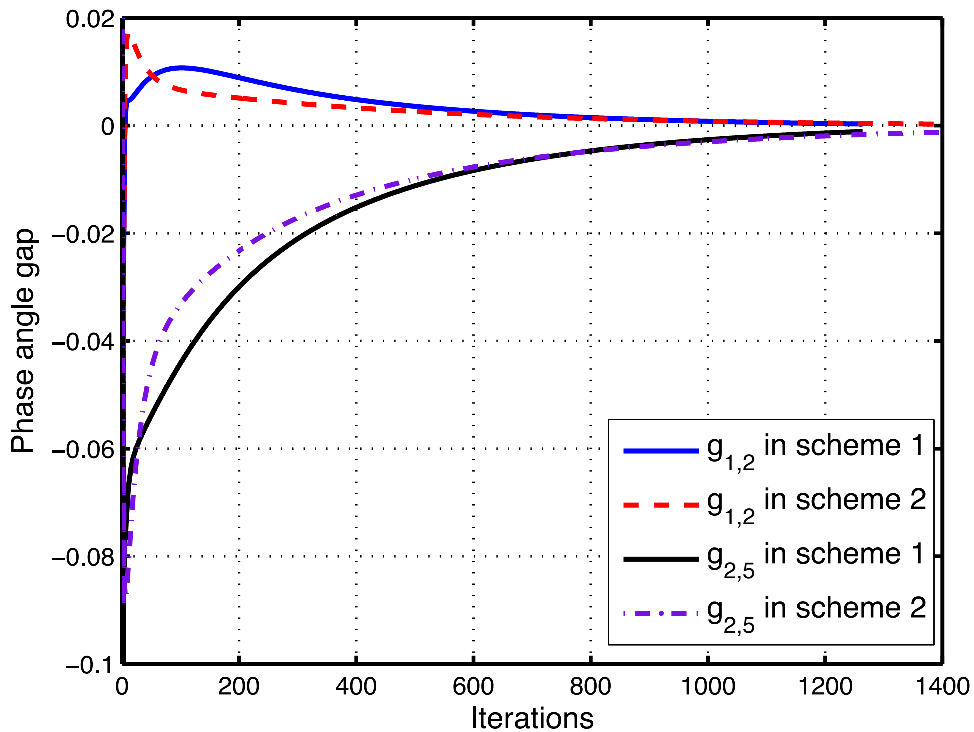


14-Bus System



118-Bus System

Communication Topology Flexibility



14-Bus System

Related Work

- Nonlinear (AC) state estimation [w/ Xie, Choi, Kar, T-SG'12]

- Multi-cast routing [w/ Li, Lai, JSAC:SG Series'12]

- Games for privacy-aware distributed state estimation [w/ Belmega, Sankar – NetGCoop'12 & T-SG (submitted)]

Summary

Three Topics in Smart Grid:

- *Game Theoretic Methods for Modeling Interactions*
- *Privacy-Utility Tradeoffs for Data Sources*
- *Distributed Algorithms for State Estimation*

Thank You!